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## BALCONY AT CORNERS: DESIGN EXAMPLE WITH BWC COUPLINGS

DESIGN EXAMPLE OF A BALCONY AT A CORNER WITH BWC COUPLINGS

## REVISIONS

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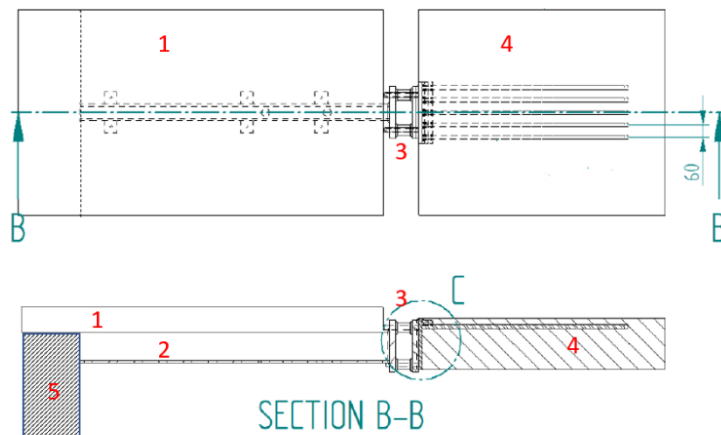
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## SUMMARY

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This report aims to give guidance on the structural design of a balcony in a corner of the building's façade, using BWC connections, manufactured by Invisible Connections AS.

The system with the BWC connection is sketched in the figure below:



*Figure 1 A concrete slab (1) resting on cantilever steel beams (2) and BWC units (3) connecting the steel beams to the concrete slab of the building (4) by means of steel bars embedded in the concrete. Optionally, a beam at the front of the balcony (5) can be built in steel or concrete.*

The recommended approach for the analysis of the structure is to use a Finite Element Analysis (FEA), where the behaviour of the BWC connection is modelled with a set of springs. The actual spring stiffness for the different degrees of freedom shall be found according to a representative force/displacement (moment/rotation) curve for the BWC-connection. Modelling the cantilevering steel beams as fully restrained at the BWC-connection will lead to unrealistic high forces, and underestimated vertical displacements. A simple example is presented to illustrate this effect.


On previous reports a capacity check of the connections is done with a 3D FEA software. This ULS capacity check doesn't include torsion. Straight beams (at a right angle with the façade) have little torsion, less than 1 kNm, and the couplings need not to be checked against it. Skewed beams (at an angle <math><90^\circ</math> from the façade), on the contrary, have torsion moments of 10 kNm or more, and a capacity check against torsion is needed.

This report proposes an assessment of the torsion capacity of the BWC connection based on a simple ULS check on the bolts according to the Eurocode.

A simple structural model is presented to assess the influence of torsion and further reduction of capacity of the BWC couplings when the beams are skewed (at an angle from the perpendicular of the building façade). The reduced capacity is evident even with small skew angles.

In section 2.4.1, a simple beam model is also presented to quickly estimate the maximum span length of the concrete slab.

Chapter 3 includes a design of a balcony based on a 100 mm prefab concrete slab with a 8x100mm



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reinforcement net, placed centric in the thin cross section. It also includes a design of the front beam and reinforcement, for the given example.

The overall conclusion is that the balcony can be satisfactory designed (for ULS and SLS) and constructed based on these premises:

- Use of BWC connections by Invisible Connections AS.
- Use of spring coefficients for the supports at the connections and a FEA model with beam and shell elements for analysis.
- Centric reinforcement with welded net.
- 100 mm concrete slab.
- Front beam in concrete or steel.
- Skewed or straight support beams.
- Maximum span lengths of 1.5 m in the concrete slab, less for skewed beams.

# TABLE OF CONTENTS

Figures .....	6
Tables8	
1 Introduction .....	9
1.1 GEOMETRY .....	9
1.1.1 Reinforcement.....	10
1.1.2 Front beams .....	11
1.1.3 Support beams .....	12
1.2 LOADS.....	13
1.3 Limit States for Design .....	13
1.3.1 ULS.....	13
1.3.2 SLS .....	13
1.3.3 Sign convention for forces in shell elements.....	13
2 STRUCTURAL MODELLING OF COUPLINGS .....	15
2.1 Spring stiffness of BWC couplings.....	16
2.2 Calculation of spring stiffnesses.....	18
2.3 Torsion in the couplings .....	18
2.4 Capacity check based on bolt capacity .....	19
2.4.1 Simple example with skewed beams .....	21
2.5 Capacity of the concrete slab.....	22
3 DESIGN EXAMPLE.....	25
3.1 Alternatives for design .....	25
3.1.1 Straight beams .....	25
3.1.2 Skewed beams.....	33
References.....	40

## FIGURES

Figure 1 A concrete slab (1) resting on cantilever steel beams (2) and BWC units (3) connecting the steel beams to the concrete slab of the building (4) by means of steel bars embedded in the concrete. Optionally, a beam at the front of the balcony (5) can be built in steel or concrete. ....	3
Figure 2 Geometry of the balcony for design .....	9
Figure 3 On site junction between the two concrete decks. The dotted area is cast after installation of the two parts. ....	10
Figure 4 Cross section of concrete deck showing placement of the reinforcement net (8x100 mm) in the middle of the slab. ....	10
Figure 5 Standard dimensions for a welded reinforcement net. ....	11
Figure 6 Illustration of a typical balcony with handrails and front beam, from [2]. ....	11
Figure 7 Horizontal slab and rectangular front beam. The reinforcement and width of the front beam (B) will be subject to design later in this report. ....	12
Figure 8 IPE240 structural and geometrical properties. ....	12
Figure 9 Convention for forces in shell elements .....	14
Figure 10 Local x and y directions .....	14
Figure 11 Plan and side view of the BWC40 U-H connection between concrete deck (left) and floor slab (right), from [2]. ....	15
Figure 12 3D view of the connection with a HUP beam, from [2]. The top of the connection is where the arrow text "Sveis E" is pointing. ....	15
Figure 13 Rotation spring stiffness $K_{yy}$ for bending moment $M_y$ . ....	16
Figure 14 Geometry of the example: a 100 mm horizontal slab (1x2.4 m) subject to self-weight and live load .....	17
Figure 15 Plan view of the vertical displacements [mm] of the example in Figure 14. With clamped supports (left) and spring stiffnesses (right). ....	17
Figure 16 Shell and beam 3D model for estimation of spring stiffness coefficients. ....	18
Figure 17 Coordinate system at the coupling. ....	19
Figure 18 Projection of a pure bending moment in the beam ( $M_{yb}$ ) to create a torsion moment ( $M_{xc}$ ) and a bending moment ( $M_{yc}$ ) in the coupling coordinate system ( $x_c, y_c$ ). ....	19
Figure 19 Section forces on the coupling and equivalent stresses on the 4 bolt-bearing groups. ....	20
Figure 20 Simple model of an infinite concrete slab with cantilever beams to investigate the influence of inclination angle $\theta$ with distance between beams, $d$ . In orange is shown the load distribution area for a single beam. ....	21
Figure 21 Concrete slab modelled as a continuous beam with $q=8.88$ kN/m. ....	22
Figure 22 Vertical displacements, shear and bending moment for $L=1$ m. ....	23
Figure 23 Plot of the maximum moment and shear force, happening at the first and end supports vs span length $L$ [m]. Maximum displacement at the end of the cantilever span is also shown. ....	23
Figure 24 Reduction of dimensioning shear forces ( $V_{max}$ ) in the vicinity of supports according to EN-1992-1-1, section 6.2.2. paragraph 3. ....	24
Figure 25 Alternative with 6 straight beams and perimeter concrete beams (120x240 m). ....	26
Figure 26 FEA linear elastic model, with beam and shell elements. ....	27
Figure 27 Reaction forces [kN] at the supports for ULS eq. 6.10.b. ....	27
Figure 28 Reaction moments [kNm] at the supports for ULS eq. 6.10.b. ....	28
Figure 29 Vertical displacements [mm] at the supports for SLS. The maximum value is 15.0 mm, less than the limit of $L/150=20$ mm. ....	28
Figure 30 Bending moments in x direction $M_x$ [kNm/m] in the slab for ULS eq. 6.10.b. ....	29
Figure 31 Vertical Shear forces $V_x$ [kN/m] in the slab for ULS eq. 6.10.b. Shear D-regions have been removed from the plot, according to recommendations from Eurocode. The peak values near the	

supports are due to the connection with the plate and have been neglected as the design of this detail is not covered here. ....	29
Figure 32 Utilization ratios for concrete compression in bending (URc).....	30
Figure 33 Utilization ratios for concrete compression due to shear (URsc). The values over 1 near the edges are very local and will be dependent on the type of connection between the support beams and the concrete slab. Some local reinforcement needs to be accounted for near those bolted connections. ....	30
Figure 34 Torsion moment [kNm] in beams. ....	31
Figure 35 Bending moment [kNm] in beams. ....	31
Figure 36 Vertical shear force [kN] in beams.....	31
Figure 37 Front beam cross section with reinforcement and connection with the concrete slab .....	32
Figure 38 Alternative with 5 straight and 2 skewed beams and a front beam. ....	33
Figure 39 FEA model of the alternative, with beam and shell elements. ....	34
Figure 40 Vertical reaction forces [kN] at the supports for ULS eq. 6.10.b. ....	34
Figure 41 Reaction moments [kNm] at the supports for ULS eq. 6.10.b. ....	35
Figure 42 Vertical displacements [mm] at the supports for SLS. The maximum value is 15.5 mm, less than the limit of 20 mm. ....	35
Figure 43 Bending moments $m_x$ [kNm/m] in the slab for ULS eq. 6.10.b, in local x direction (parallel to the building). ....	36
Figure 44 Vertical shear forces $V_x$ [kN/m] in the slab for ULS eq. 6.10.b. Shear D-regions have been removed from the plot, according to recommendations from Eurocode. The peak values near the supports are due to the connection with the plate and have been neglected, some local reinforcement needs to be placed at those points. ....	37
Figure 45 Utilization ratios for concrete compression in bending (URc).....	37
Figure 46 Utilization ratios for concrete compression due to shear (URsc). The values over 1 are due to the local shear concentrations on the skewed beams near the couplings that cannot be analyzed with plate theory.....	38
Figure 47 Torsion moment [kNm] in support beams.....	38
Figure 48 Bending moment [kNm] in support beams.....	39
Figure 49 Vertical shear force [kN] in support beams. ....	39

## TABLES

Table 1 Loads and Load factors for ULS and SLS.....	13
Table 2 Maximum capacity of coupling for ULS, from [2].....	16
Table 3 Bending moment ( $M_y$ ), shear force ( $V_z$ ) and torsion moment ( $M_t$ ) at the supports and maximum distance between supports ( $d_{max}$ ), with varying inclination horizontal angle $\theta$ .....	21
Table 4 Maximum shear forces ( $V_{z,max}$ ), bending moments ( $M_{y,max}$ ) and displacement ( $\delta_{max}$ ) with varying length parameter $L$ . Utilization ratios are also shown: $UR_c$ is the utilization of concrete in compression, $UR_r$ is the utilization of the longitudinal reinforcement, $UR_{sc}$ is the utilization of the concrete compression in shear and $UR_{st}$ is the utilization of the tensile stress of the concrete in shear. In red is the overutilization in the case of $L \geq 1.75$ m.....	24



# 1 INTRODUCTION

Invisible Connections AS (IC) is a provider of couplings (by the name of BWC) for cantilever prefabricated balconies. The capacity of these couplings has been analyzed in other reports but focusing on a straight geometry. The focus of this report is to investigate on optimization of alternatives and analysis strategies for an example case with a balcony at a corner around the building's façade.

## 1.1 GEOMETRY

The geometry of the example case is shown below:

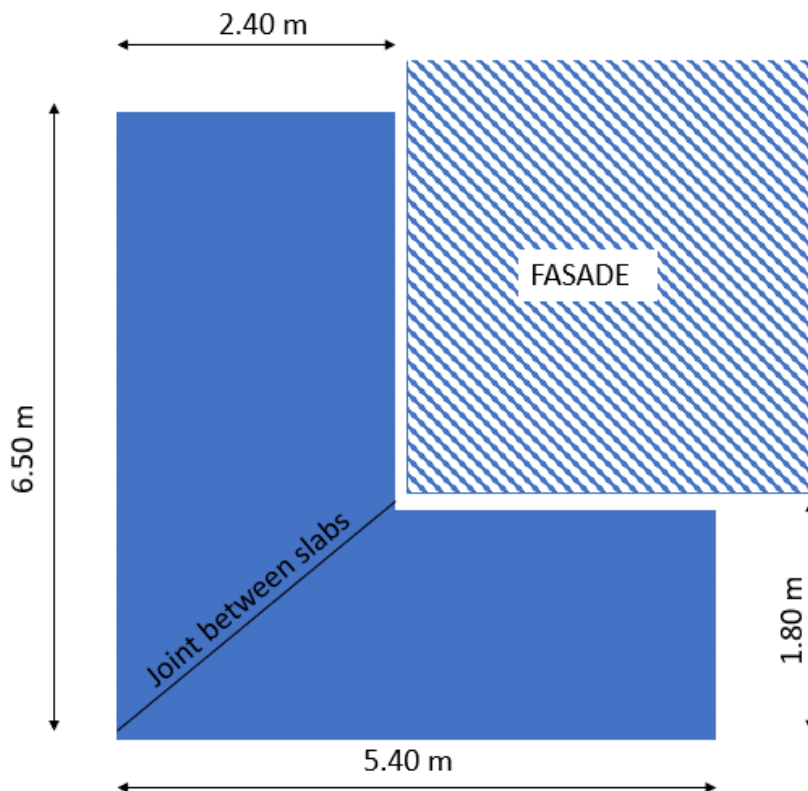


Figure 2 Geometry of the balcony for design

The balcony has two parts that are prefabricated and connected along the black line in Figure 2 to avoid differential deformation. This joint can be made with a shear-key in concrete, as illustrated in Figure 3. Such a connection will not transmit moment.

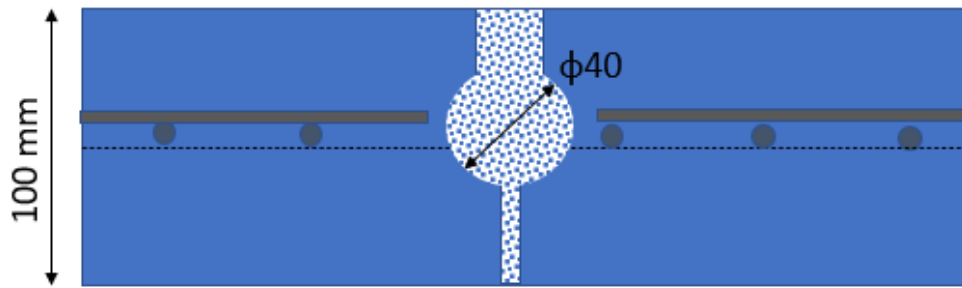


Figure 3 On site junction between the two concrete decks. The dotted area is cast after installation of the two parts.

### 1.1.1 Reinforcement

The balcony has a 100 mm C35 concrete slab with reinforcement based on a 8x100 mm welded net of B500NA bars. We have seen there is a predominance of negative moments (that is, with tension on the upper half of the cross section) over the positive, which recommends that the reinforcing net is located in the upper part of the cross section just above the center. The reinforcement bars will therefore not lie exactly in the middle of the slab but will be eccentrically positioned. The bars in x direction will be at a distance of 38 mm from the top side, see Figure 4.

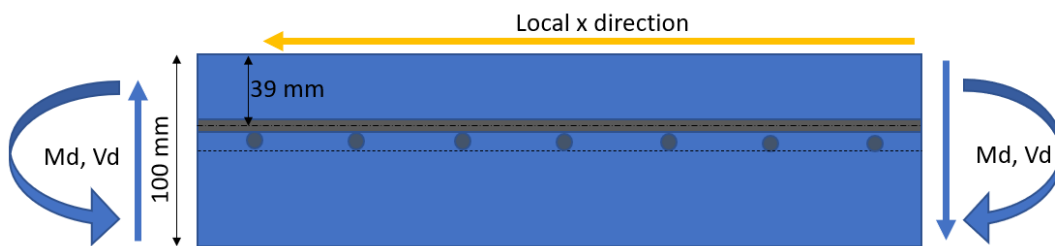


Figure 4 Cross section of concrete deck showing placement of the reinforcement net (8x100 mm) in the middle of the slab.

The slab will work mainly in the direction parallel to the building (the x direction defined later in this report), therefore it is recommended to lay the net with the bars in this direction placed higher in the cross section to optimize it against negative moments, as shown in Figure 4.

For completeness, the available standard dimensions for reinforcement nets in Norway are shown in the table below:

## Armeringsnett B500NA

NS 3576-1

Type	Format m	Tråd mm	Masker mm	Vekt		
				kg	stk/pk	vekt/pk
P091	2,0 x 5,0	3,4	100x100	14,2	70	994
P091S	1,2 x 2,4	4	100x100	4,1	50	205
K131	2,0 x 5,0	5	150x150	21	50	1050
K131S	1,2 x 2,4	5	150x150	5,91	50	295
K189	2,0 x 5,0	6	150x150	30,2	40	1208
K257	2,0 x 5,0	7	150x150	41,1	30	1233
K335	2,0 x 5,0	8	150x150	53,7	20	1074
K402	2,0 x 5,0	8	125x125	66	15	990
K503	2,0 x 5,0	8	100x100	79	15	1185

Figure 5 Standard dimensions for a welded reinforcement net.

### 1.1.2 Front beams

It is normal to have a front beam along the outer sides to conceal the cantilever beams that carry the balcony. The handrail can also be connected to the front beam. Figure 6 illustrates a typical balcony.



Figure 6 Illustration of a typical balcony with handrails and front beam, from [1].

The beams supporting the balcony are IPE240 steel beams (S355) and the perimeter beam cross section will have a height of 240 mm to cover the beams from sight.

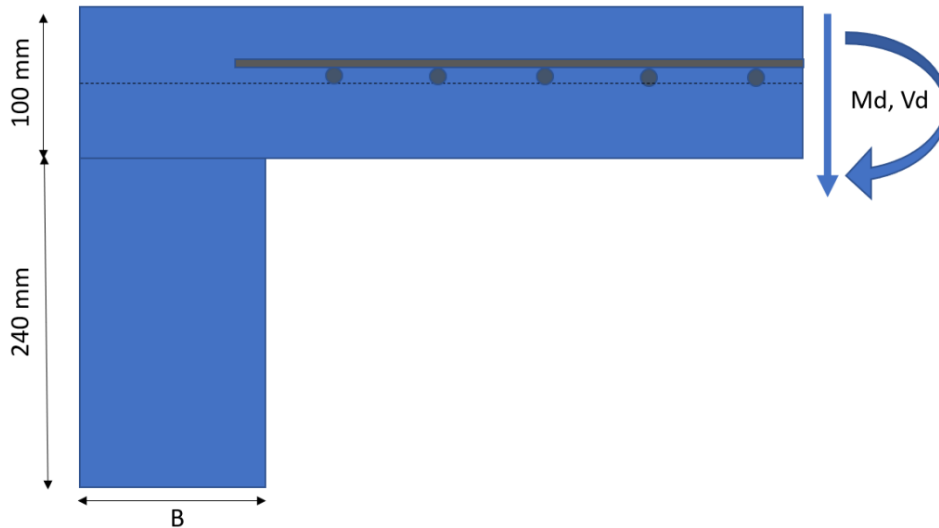
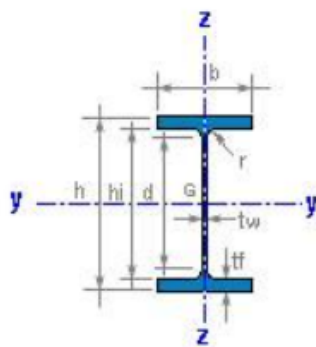


Figure 7 Horizontal slab and rectangular front beam. The reinforcement and width of the front beam (B) will be subject to design later in this report.

The minimum nominal concrete cover will be 35 mm.

### 1.1.3 Support beams

#### IPE 240



$h = 240 \text{ mm}$	$r = 15 \text{ mm}$
$b = 120 \text{ mm}$	$d = 190.4 \text{ mm}$
$tw = 6.2 \text{ mm}$	$hi = 220.4 \text{ mm}$
$tf = 9.8 \text{ mm}$	
$A = 39.1 \text{ cm}^2$	$M = 30.7 \text{ kg/m}$
$I_y = 3892 \text{ cm}^4$	$I_z = 284 \text{ cm}^4$
$W_y = 324.3 \text{ cm}^3$	$W_z = 47.3 \text{ cm}^3$
$W_{ply} = 366.7 \text{ cm}^3$	$W_{plz} = 73.9 \text{ cm}^3$
$i_y = 9.97 \text{ cm}$	$i_z = 2.69 \text{ cm}$
$I_t = 13.0 \text{ cm}^4$	$I_w = 37576 \text{ cm}^6$
$S_y = 183.3 \text{ cm}^3$	$Av_z = 19.15 \text{ cm}^2$
$s_y = 21.2 \text{ cm}$	
$AL = 0.922 \text{ m}^2/\text{m}$	$AG = 30.02 \text{ m}^2/\text{t}$

Figure 8 IPE240 structural and geometrical properties.

## 1.2 LOADS

The loads are in accordance with Eurocode (EN 1991-1-1), they are shown in the table below:

Table 1 Loads and Load factors for ULS and SLS.

Load	Characteristic Value	Type Load	Load factor ULS	Load factor SLS
Self-weight-concrete	24 kN/m <sup>3</sup>	Permanent	1.35/1.2	1
Self-weight-steel	78.5 kN/m <sup>3</sup>	Permanent	1.35/1.2	1
Handrail	1 kN/m	Permanent	1.35/1.2	1
Live Load (Table 6.2 EC1+NA)	4 kN/m <sup>2</sup>	Variable	1.05/1.5	0.6

Load factor = $\xi \cdot \gamma_{G,i}$  (PERMANENT LOADS) or  $q_{i,i} \cdot \psi_{0,i}$  (VARIABLE LOADS) given for Eq 6.10a/Eq 6.10b

## 1.3 Limit States for Design

### 1.3.1 ULS

The BWC couplings between the IPE beams and the building are the weakest link and will therefore always govern the design. The IPE beams are governed by SLS design. The capacity of the concrete part (reinforcement and concrete) will also be checked.

### 1.3.2 SLS

The main check in the Service Limit State is the vertical deformation of the concrete slab. The deformation should be less than  $L/150$ , where  $L$  is the length of the cantilever beam. At the corner the deflection limit is about  $3m/150=20$  mm.

### 1.3.3 Sign convention for forces in shell elements

The bending moments  $M_y$  and  $M_x$ , torsion moments  $M_{xy}$ ,  $M_{yx}$  and shear for shell elements follow the convention shown in the figure below:

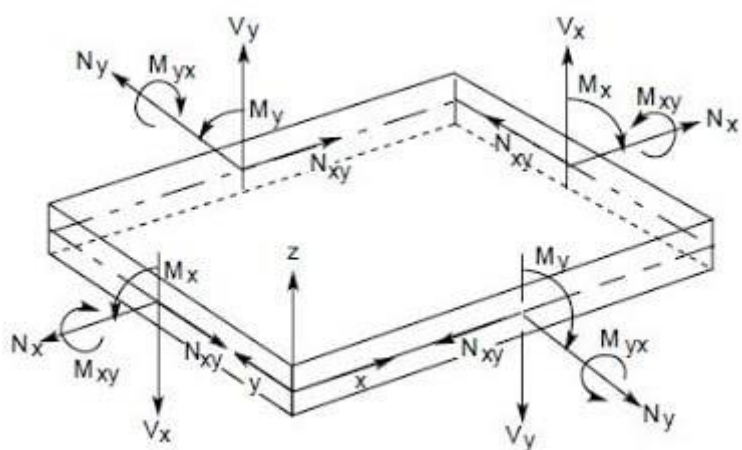


Figure 9 Convention for forces in shell elements

The x direction follows the building façade, as shown in the picture below. Y direction is perpendicular to x in the horizontal plane, and z direction points to the ground:

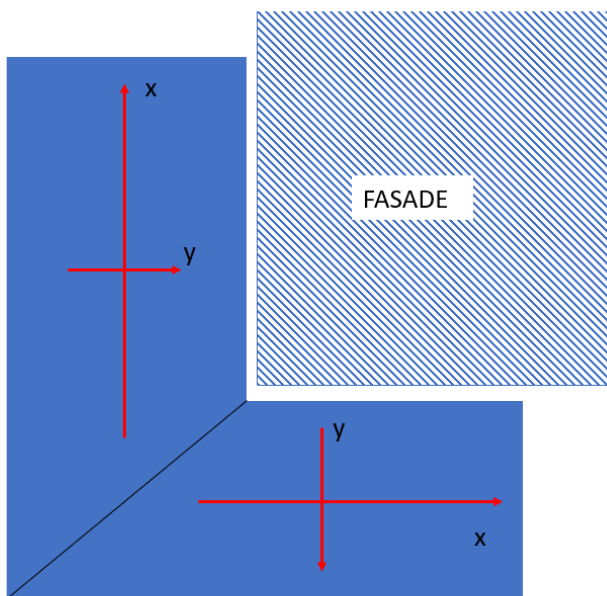


Figure 10 Local x and y directions

## 2 STRUCTURAL MODELLING OF COUPLINGS

The BWC couplings structurally connect the beams supporting the balcony slab with the floor slab in the building structure.

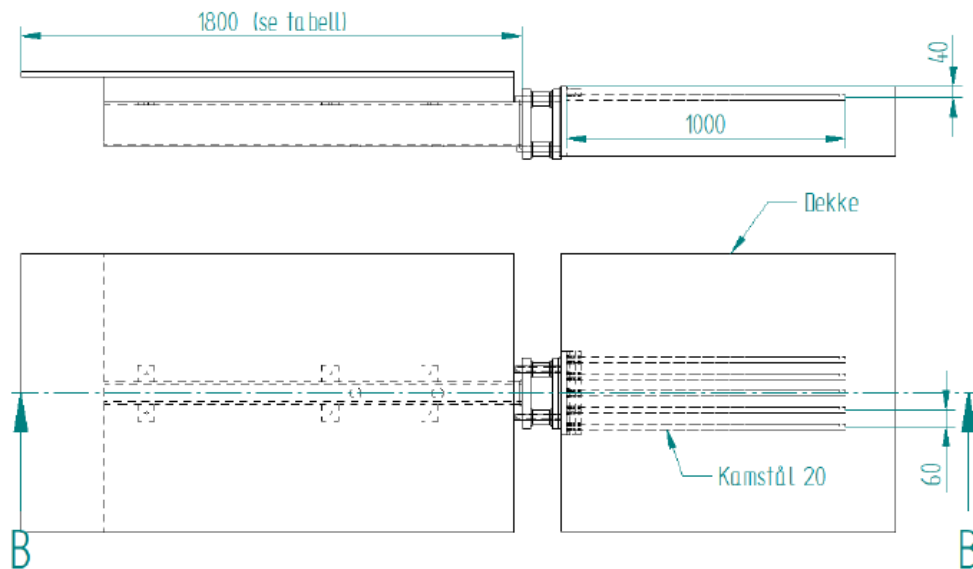


Figure 11 Plan and side view of the BWC40 U-H connection between concrete deck (left) and floor slab (right), from [1].

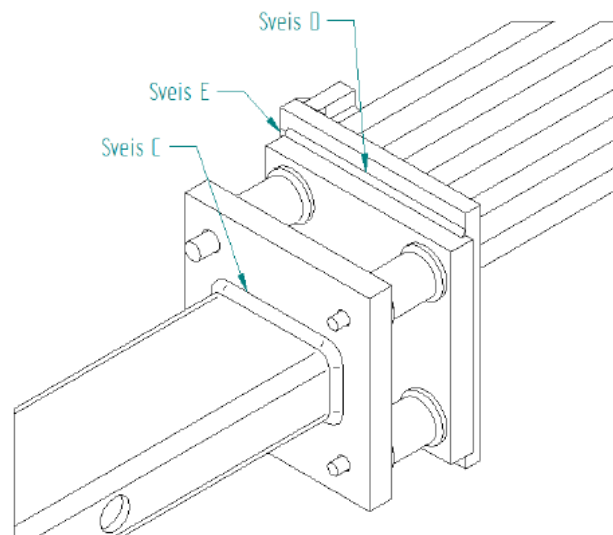


Figure 12 3D view of the connection with a HUP beam, from [1]. The top of the connection is where the arrow text "Sveis E" is pointing.

The BWC couplings have M24 bolts in the two upper positions and M16 bolts on the lower positions. The bearings are assumed to have an 60x5mm annular cross section.

The bearing capacity of the BWC40 U-H connection has been determined in [1], based on an ANSYS 3D FEA model:

Table 2 Maximum capacity of coupling for ULS, from [1].

Coupling	Max. bending moment [kNm]	Max. shear force [kN]	Max. axial force [kN]
BWC 40 U-H	60	70	+/-20

## 2.1 Spring stiffness of BWC couplings

The balcony slab rests on top of the steel beams. The loads over the deck will be transferred to the steel beams and further to the couplings. A common practice is to assume that the beams are clamped to the building, but this leads to unrealistic and very high concentrated loads on the couplings and underestimates the vertical displacements. It is therefore recommended that spring stiffnesses are used to model the supports both in ULS and SLS analysis. This will lead to a more even distribution of forces and allow for a higher total capacity of all the couplings.

Modelling boundary conditions with springs is a common simplification of flexible supports. This tool is supported by most Finite Element software. It allows to create a flexible support with reactions that will depend on the deformations, just like in a regular spring.

The spring stiffness is the relation between the action at the support and the deformation measured at support. In the figure below, an example with the bending moment is shown. The bending stiffness coefficient of the support is  $K_{yy} = M_y / \varphi_y$ , where  $M_y$  is the bending moment at the support and  $\varphi_y$  is the corresponding rotation angle in the same direction as  $M_y$ , measured at the support as sketched in Figure 13.

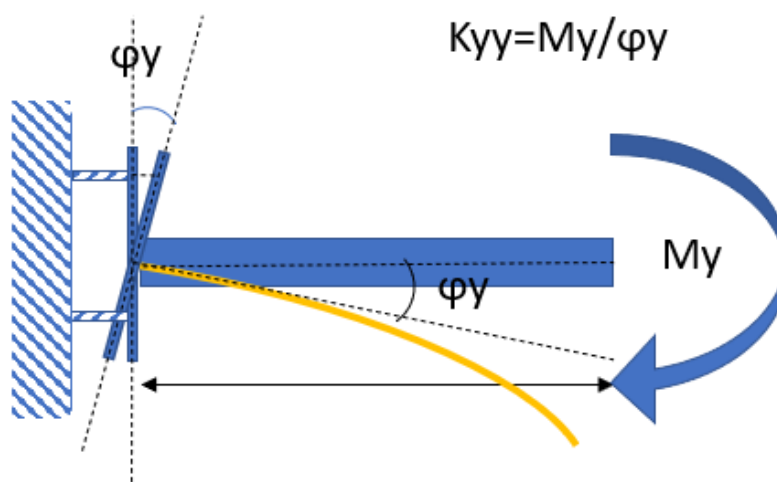


Figure 13 Rotation spring stiffness  $K_{yy}$  for bending moment  $M_y$ .



Spring stiffnesses for loads in other directions (N, Vy, Vz, Mx and Mz) can be defined in a similar manner.

$$K_x = \frac{N}{u_x} \text{ [kN/m]}$$

$$K_y = \frac{V_y}{u_y} \text{ [kN/m]}$$

$$K_z = \frac{V_z}{u_z} \text{ [kN/m]}$$

$$K_{xx} = \frac{M_x}{\varphi_x} \text{ [kN/rad]}$$

$$K_{yy} = \frac{M_y}{\varphi_y} \text{ [kN/rad]}$$

$$K_{zz} = \frac{M_z}{\varphi_z} \text{ [kN/rad]}$$

The advantage of this approach is that one can model any geometry, beam cross section or configuration of supports with a simple matrix structural program with beams and shell elements. The difference in behaviour with clamped supports can be observed in the deformations of a simple model with a concrete plate supported by two IPE240 beams, see Figure 15.

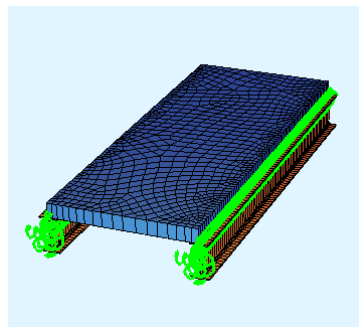


Figure 14 Geometry of the example: a 100 mm horizontal slab (1x2.4 m) subject to self-weight and live load

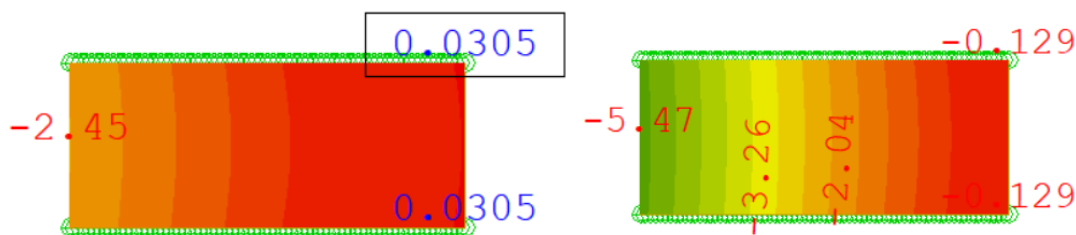
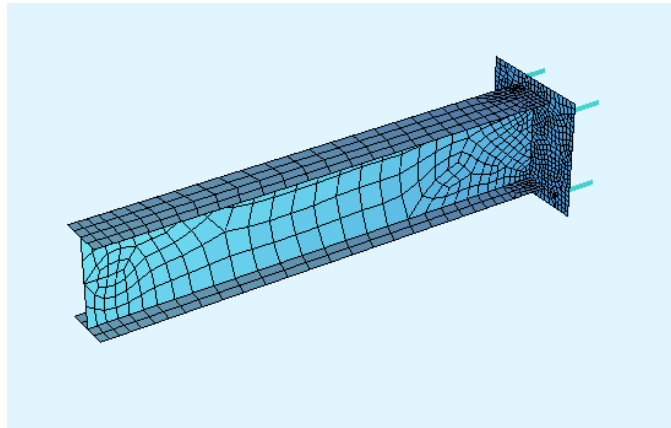


Figure 15 Plan view of the vertical displacements [mm] of the example in Figure 14. With clamped supports (left) and spring stiffnesses (right).

The reactions would also be different in both models if we were in a statically indeterminate structure.

## 2.2 Calculation of spring stiffnesses

There is still some work to be done to ensure the best way to estimate the spring coefficients. In this report we have used a 3D model containing shell (front plate and steel beam (IPE240)) and beam (bolts) elements to obtain approximate values, see Figure 16.



*Figure 16 Shell and beam 3D model for estimation of spring stiffness coefficients.*

## 2.3 Torsion in the couplings

Beams are mainly subject to bending moment, therefore torsion is not seen as a problem if the beams are straight (that is, forming a 90° angle with the façade), with maximum values that can be in the range of 1-2 kNm. With skewed beams values near 10 kNm or greater can appear in the BWC coupling. This is mainly due to the projection of the bending moment of the skewed beam, which gives a component as torsion in the coupling coordinate system, see Figure 18.

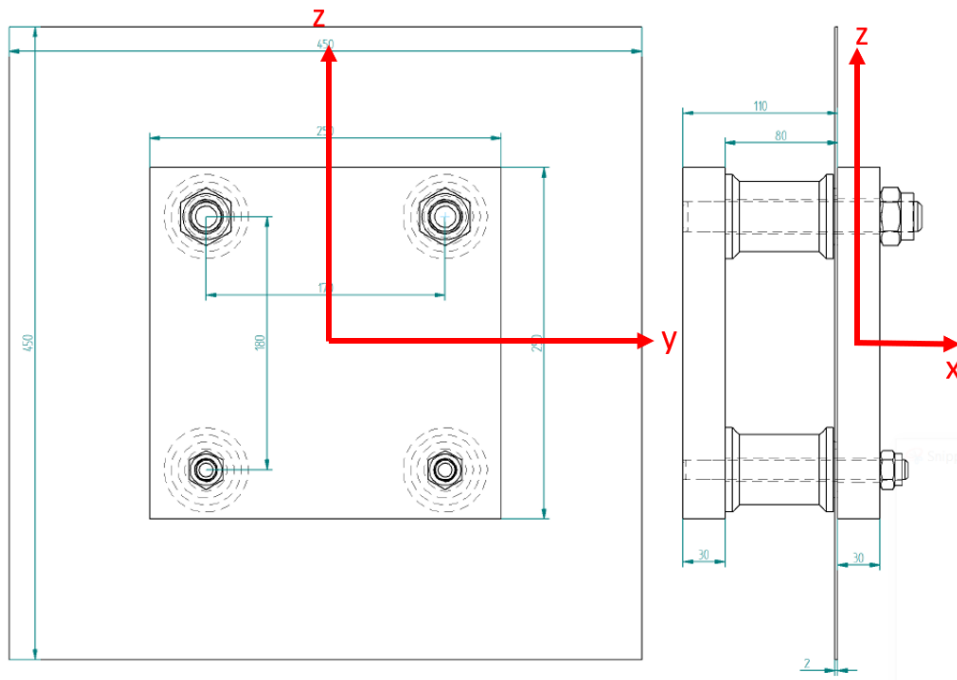


Figure 17 Coordinate system at the coupling.

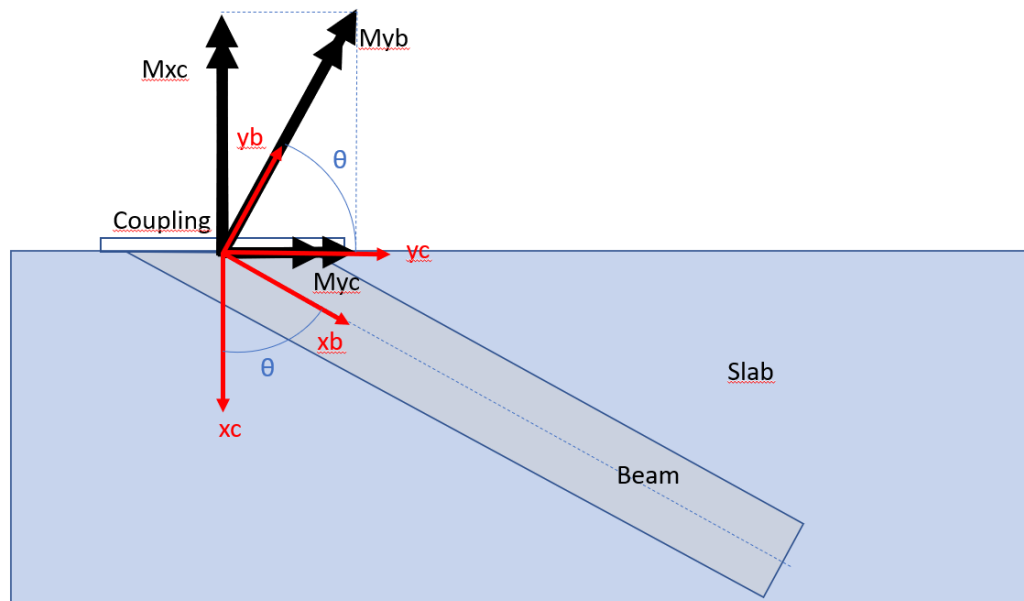


Figure 18 Projection of a pure bending moment in the beam ( $M_{yb}$ ) to create a torsion moment ( $M_{xc}$ ) and a bending moment ( $M_{yc}$ ) in the coupling coordinate system ( $x_c, y_c$ ).

## 2.4 Capacity check based on bolt capacity

The capacity of the coupling against bending moment combined with shear and torsion has not been assessed yet. It can be estimated by checking the capacity of the bolts of the connection, particularly on the two upper bolts under tension. The distribution of shear and normal stresses in the bolts from the reactions on the front plate ( $M_y, V_z, M_x$ ) is obtained by means of a simple distribution of forces based on the stiffness (cross sectional area) of the bolts. In the case of normal stresses, the combined

area of the bolts and the bearings is used, and in the case of shear, the area of the bolts alone. By this we are assuming the bearings do not carry shear, being carried only by the bolts.

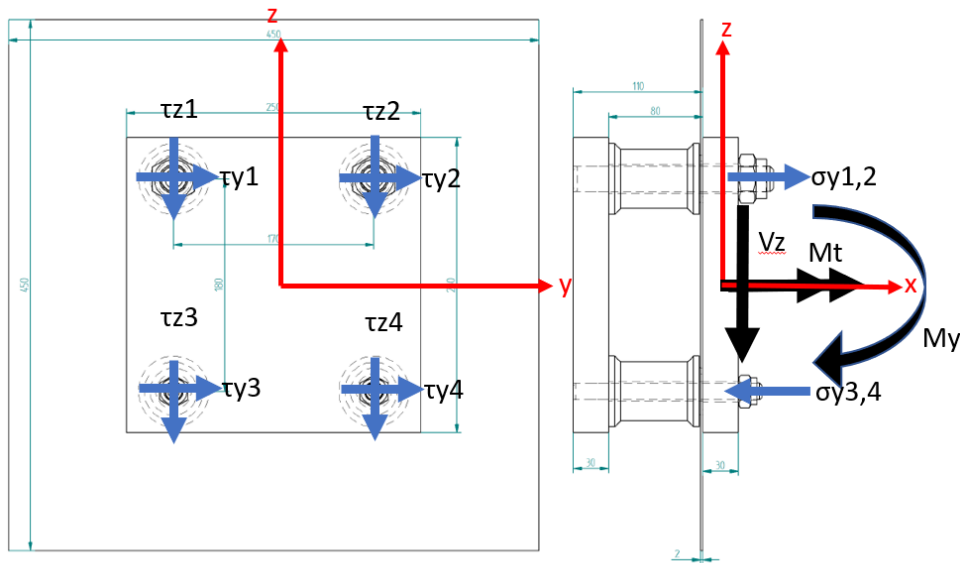


Figure 19 Section forces on the coupling and equivalent stresses on the 4 bolt-bearing groups.

The upper bolts are preloaded with a torque value of 735 Nm, which is assumed to give a 150 kN preload force in the bolts, ref [1]. Any axial load on the bolt-bearing group will therefore be distributed between the bolt and the bearing just until the bearing rings are no longer in compression and all the load is taken by the tension stress in the bolts. This point is reached when the upper bolt-bearing group is loaded with approximately 200 kN of tension load. This hardly happens, since it would require a bending moment of approximately 72 kNm. The shear load is assumed to be taken by the bolts alone, distributed according to the bolt area.

The capacity check on the bolts is made according to [2], Section 3.6 for bolts with tension and shear combined:

Combined shear and tension	$\frac{F_{v,Ed}}{F_{v,Rd}} + \frac{F_{t,Ed}}{1,4 F_{t,Rd}} \leq 1,0$
----------------------------	--

With the tension and shear resistance ( $F_{v,Rd}$  and  $F_{t,Rd}$ ) given from the equations below:

**TENSION**

Failure mode	Bolts	Rivets
Shear resistance per shear plane	$F_{v,Rd} = \frac{\alpha_v f_{ub} A}{\gamma_{M2}}$ <ul style="list-style-type: none"> <li>- where the shear plane passes through the threaded portion of the bolt (<math>A</math> is the tensile stress area of the bolt <math>A_s</math>):</li> <li>- for classes 4.6, 5.6 and 8.8: <math>\alpha_v = 0,6</math></li> <li>- for classes 4.8, 5.8, 6.8 and 10.9: <math>\alpha_v = 0,5</math></li> <li>- where the shear plane passes through the unthreaded portion of the bolt (<math>A</math> is the gross cross section of the bolt): <math>\alpha_v = 0,6</math></li> </ul>	$F_{v,Rd} = \frac{0,6 f_{ur} A_0}{\gamma_{M2}}$

Tension resistance <sup>2)</sup>	$F_{t,Rd} = \frac{k_2 f_{ub} A_s}{\gamma_{M2}}$ <p>where <math>k_2 = 0,63</math> for countersunk bolt, otherwise <math>k_2 = 0,9</math>.</p>
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### 2.4.1 Simple example with skewed beams

With the method for estimating the capacity of the connection mentioned above and a simple cantilever beam model we have investigated the effect of the inclination angle on the maximum capacity of the connections.

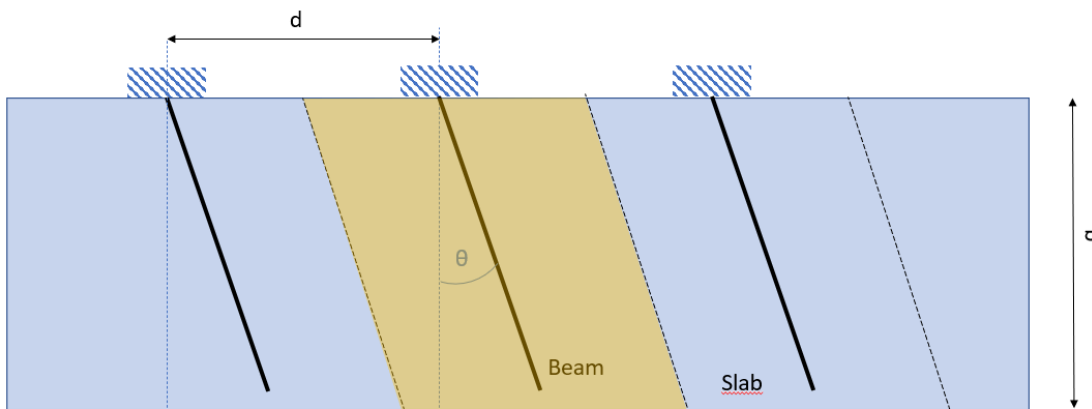


Figure 20 Simple model of an infinite concrete slab with cantilever beams to investigate the influence of inclination angle  $\theta$  with distance between beams,  $d$ . In orange is shown the load distribution area for a single beam.

With a fixed width of  $b=2.4$  m (consistent with the example case below), the loads described in 1.2 and the bolt capacity check described above according to EC, we have investigated the maximum distance ( $d_{max}$ ) which leads to overutilization of the upper bolts for different values of  $\theta$  for ULS (eq. 6.10b). The results are shown in the table below:

Table 3 Bending moment ( $M_y$ ), shear force ( $V_z$ ) and torsion moment ( $M_t$ ) at the supports and maximum distance

between supports ( $d_{max}$ ), with varying inclination horizontal angle  $\theta$ .

$\theta$ [°]	My [kNm]	Vz [kNm]	Mt [kNm]	dmax [m]
0	65.4	50.2	0.0	2.3
5	59.8	46.0	5.2	2.1
10	42.7	33.2	7.5	1.5
15	31.3	24.6	8.4	1.1
20	25.6	20.4	9.3	0.9
25	19.9	16.1	9.3	0.7
30	17.1	14.0	9.9	0.6
35	14.2	11.9	10.0	0.5
40	11.4	9.7	9.6	0.4
45	8.5	7.6	8.5	0.3

Greater angles than 45 ° are not seen as practical, a very wide  $\theta$  angle can also lead to undesirable local effects at the beam-coupling connection.

Although the model is quite simple, we can see that the effect of the inclination of the beams is significant, especially for lower angles (5°-15°).

The limitations of this model are mainly that, on the one hand the nearby beams will take the loads from the ones that are more loaded as this is not a statically determined system, and on the other hand the beams are not fully clamped, which again will help to distribute the loads more evenly among the couplings, as stated in 2.1. The model also considers the same inclination for all the beams, different inclinations between the beams will give uneven distribution of reactions and therefore higher peak values.

A more thorough and detailed calculation is shown in the example case in section 3.

## 2.5 Capacity of the concrete slab

The concrete slab can be seen as a continuous beam with a width of 1 m (neglecting the effect of the flexible supports and the perimeter beam). We can then parameterize the maximum moment and shear force in the beam by considering 5 spans of length L, with a total factored load  $q=1.2*24\text{kN/m}^3*0.1\text{ m}^3+1.5*4\text{kN/m}^2*1\text{ m}=8.88\text{ kN/m}$



Figure 21 Concrete slab modelled as a continuous beam with  $q=8.88\text{ kN/m}$ .

One can further calculate the utilization of the concrete slab (concrete compression, reinforcement

and concrete tensile and compression strength in shear). This gives us the results presented in the figures below:

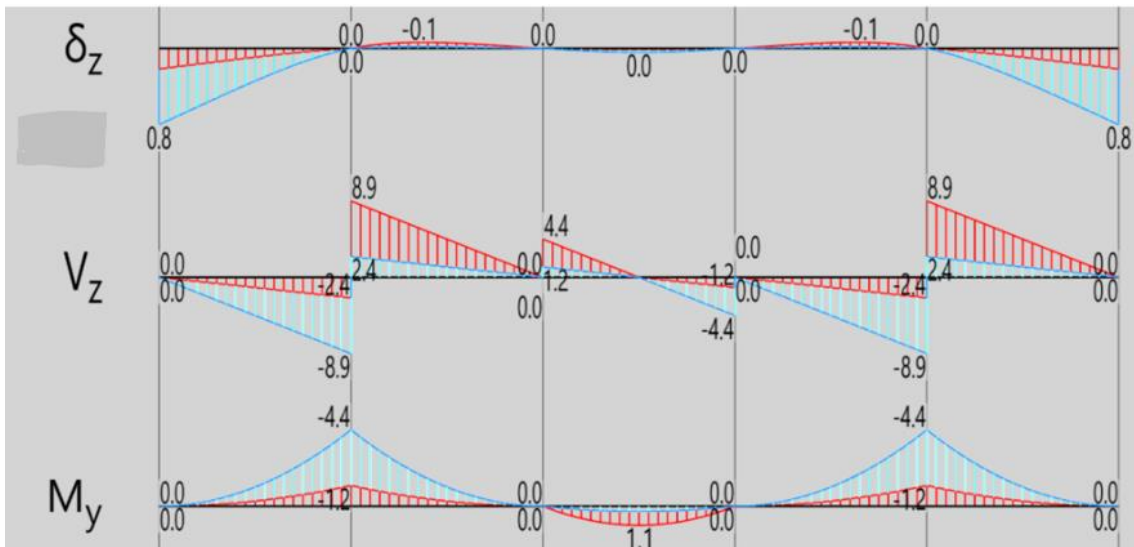


Figure 22 Vertical displacements, shear and bending moment for  $L=1$  m.

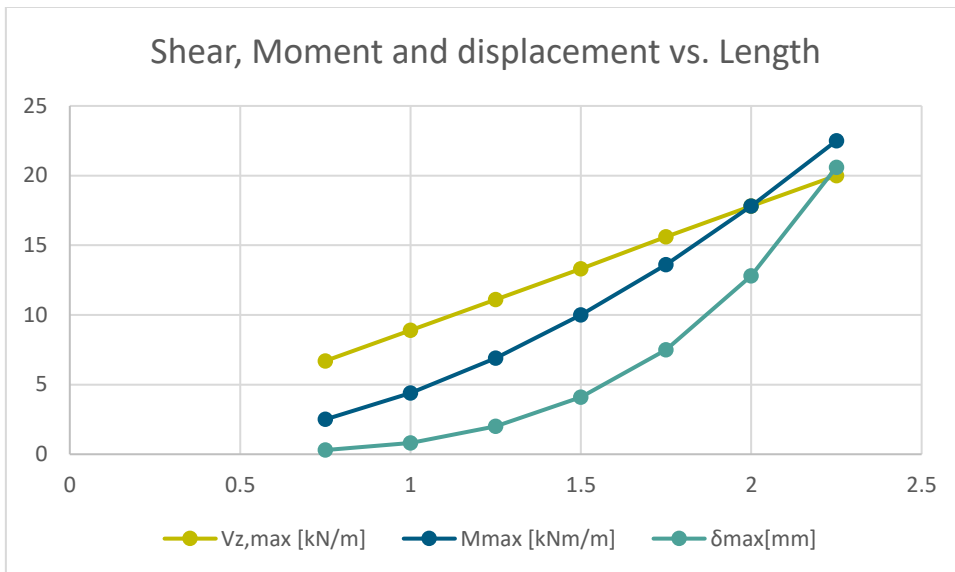


Figure 23 Plot of the maximum moment and shear force, happening at the first and end supports vs span length  $L$ [m]. Maximum displacement at the end of the cantilever span is also shown.

Table 4 Maximum shear forces ( $V_{z,max}$ ), bending moments ( $M_{y,max}$ ) and displacement ( $\delta_{max}$ ) with varying length parameter  $L$ . Utilization ratios are also shown:  $UR_c$  is the utilization of concrete in compression,  $UR_r$  is the utilization of the longitudinal reinforcement,  $UR_{sc}$  is the utilization of the concrete compression in shear and  $UR_{st}$  is the utilization of the tensile stress of the concrete in shear. In red is the overutilization in the case of  $L \geq 1.75$  m.

L [m]	$V_{z,max}$ [kN/m]	$M_{y,max}$ [kNm/m]	$\delta_{max}$ [mm]	$UR_c$	$UR_r$	$UR_{sc}$	$UR_{st}$
0.75	6.7	2.5	0.3	0.218	0.211	0.034	0.177
1	8.9	4.4	0.8	0.373	0.372	0.045	0.235
1.25	11.1	6.9	2	0.561	0.584	0.056	0.293
1.5	13.3	10	4.1	0.762	0.85	0.068	0.352
1.75	15.6	13.6	7.5	1.129	1.13	0.079	0.412

The utilisation ratios are calculated based on the reinforcement given in section 1.1.1.

The real cross section forces in the real structure are assumed to be higher due to 3D effects and local concentration of forces. Though this simple model allows for a good start point when choosing the spans of the concrete slab.

It is also worth explaining that the cross-section forces are dependent on the type of connection between IPE beams and slab. We have chosen an elastic link between the two.

It is also important to remark that, according to Eurocode, the dimensioning shear force ( $V_{max}$ ) should be considered at a distance  $d$  from the edge of the supports. In our case that would be at a distance  $100+60=160$  mm from each beam centerline.

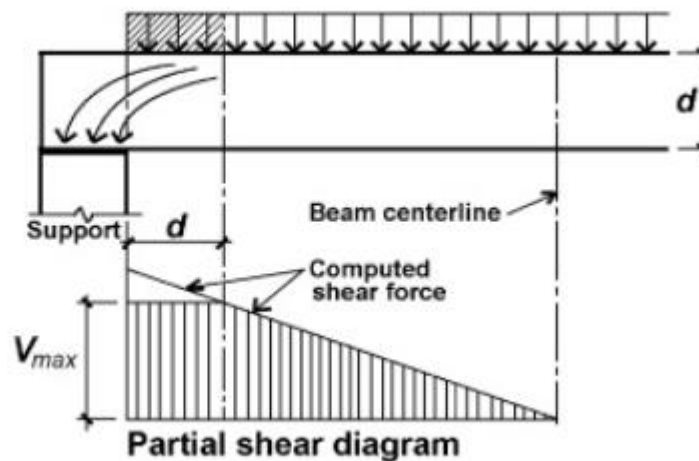


Figure 24 Reduction of dimensioning shear forces ( $V_{max}$ ) in the vicinity of supports according to EN-1992-1-1, section 6.2.2. paragraph 3.



## 3 DESIGN EXAMPLE

In this section two design examples are presented based on the geometry and loads presented in sections 1.1 and 1.2. Spring stiffness coefficients have been used for the supports as explained in 2.2.

3D FEA models have been developed for both alternatives with Sofistik software for analysis.

### 3.1 Alternatives for design

Two main configurations have been analyzed with the same overall dimensions: one with skewed beams, and another with straight beams, both have a front beam in concrete along the outer sides of the slab.

#### 3.1.1 Straight beams

This solution places the BWC's at a 90-degree angle to the façade. This introduces a large cantilever for the balcony deck towards the corner. To strengthen and stiffen this cantilever we utilize the capacity in the front beam.

##### 3.1.1.1 Geometry

The layout of beams and the geometry is shown in the figure below:

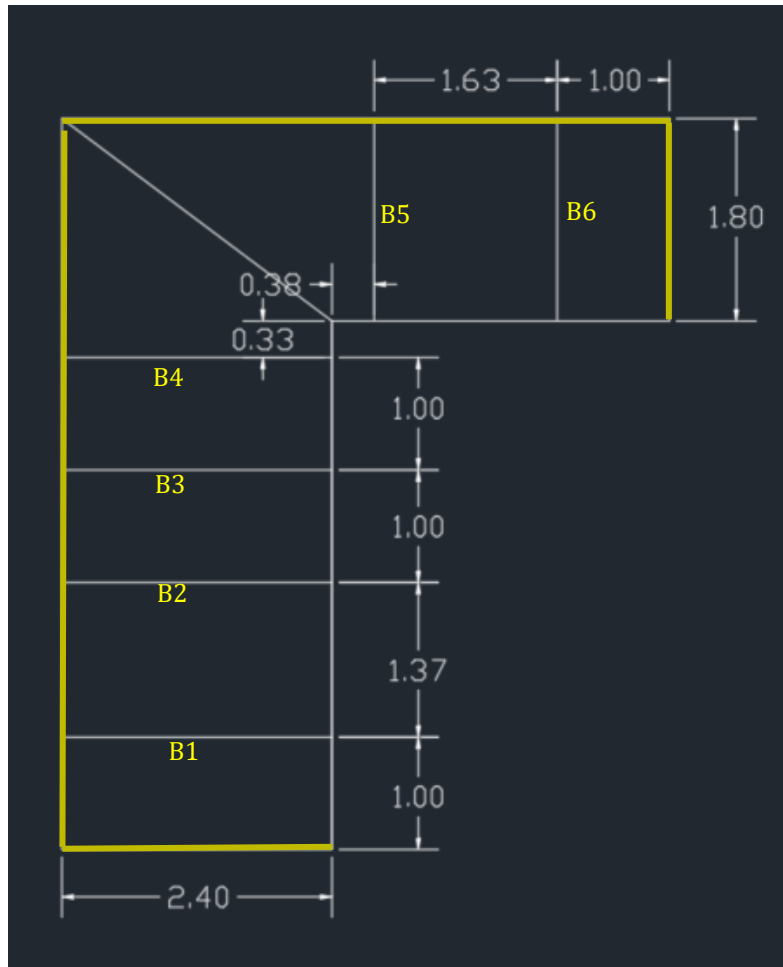


Figure 25 Alternative with 6 straight beams and perimeter concrete beams (120x240 m).

The reinforcement of the balcony slab is as described in section 1.1.1.

The front beams can also be made of steel with the same cross section as the other supporting beams (IPE240). This can affect the composite action between the slab and the beams, depending on the connection between the two, especially for the transfer of moments. The design of the front beams is discussed later in section 3.1.1.5.

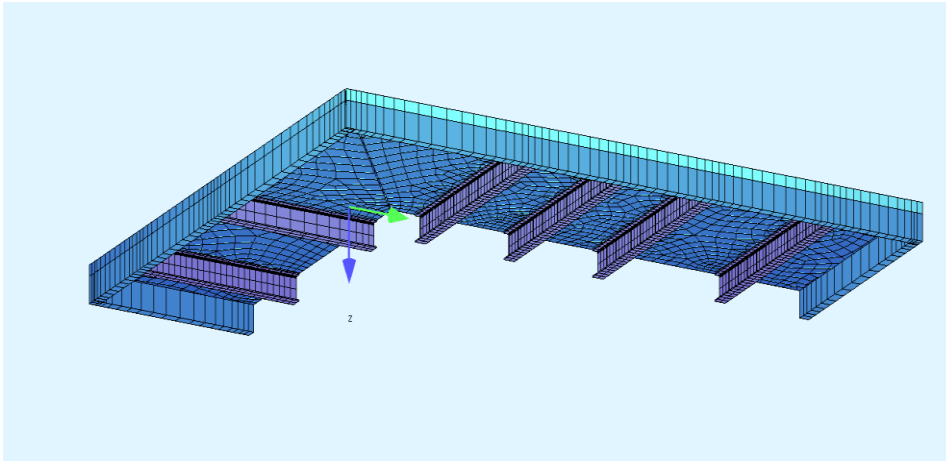


Figure 26 FEA linear elastic model, with beam and shell elements.

### 3.1.1.2 Results

For ULS (eq. 6.10b):

The reactions at the supports (couplings) are as shown in the figure below:

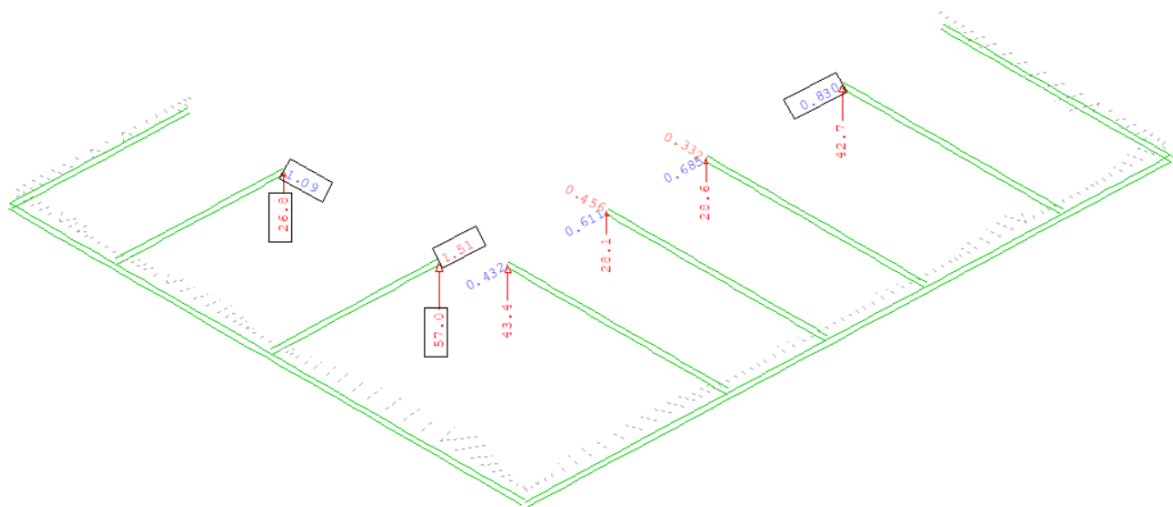


Figure 27 Reaction forces [kN] at the supports for ULS eq. 6.10.b.

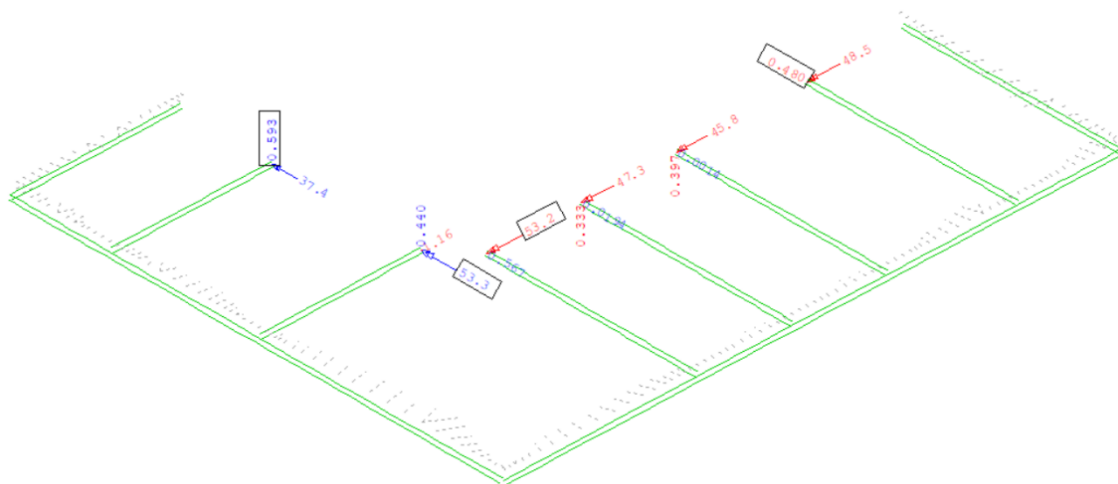


Figure 28 Reaction moments [kNm] at the supports for ULS eq. 6.10.b.

All the reactions are in the range of  $M_y < 60$  kNm and  $V_z < 70$  kN, set as limit for ULS. The maximum torsion moment is  $M_x = 1.16$  kNm corresponding to the support of beam B5 ( $M_y = 53.3$  kNm and  $V_z = 57.0$  kN), which gives a **utilization ratio of 0.98** in the upper bolts, according to the capacity check described in 2.3.

For SLS:

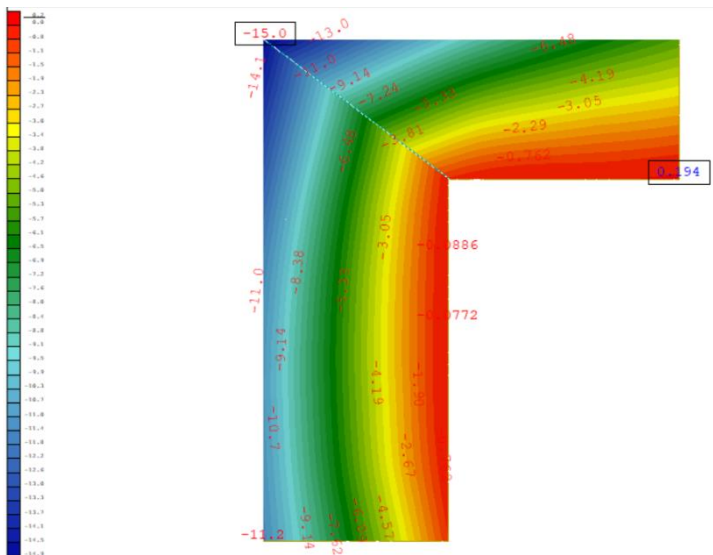


Figure 29 Vertical displacements [mm] at the supports for SLS. The maximum value is 15.0 mm, less than the limit of  $L/150 = 20$  mm.

### 3.1.1.3 Concrete slab.

The concrete slab is checked against bending moments and shear. Figure 30 and Figure 31 show the distribution of maximum moments and shear forces in the direction of the façade.

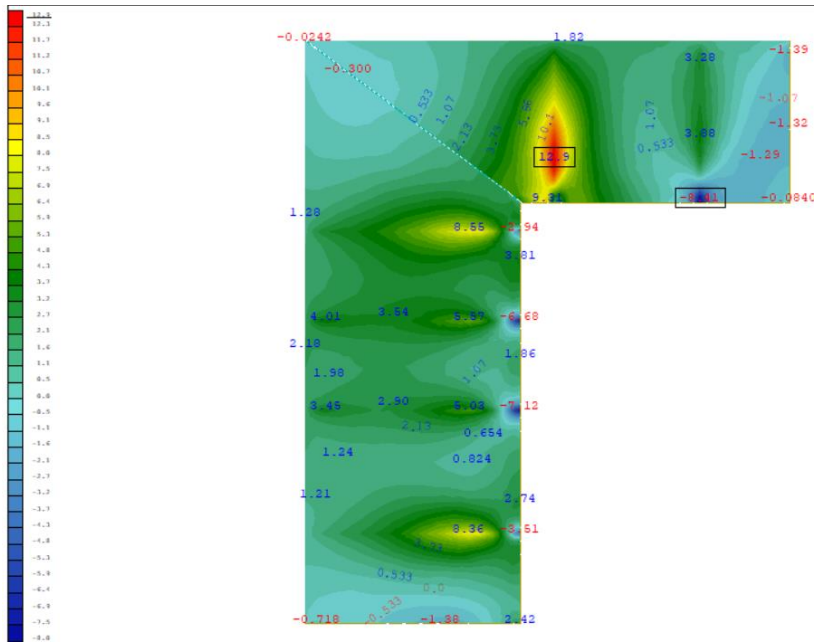


Figure 30 Bending moments in x direction  $M_x$  [kNm/m] in the slab for ULS eq. 6.10.b.

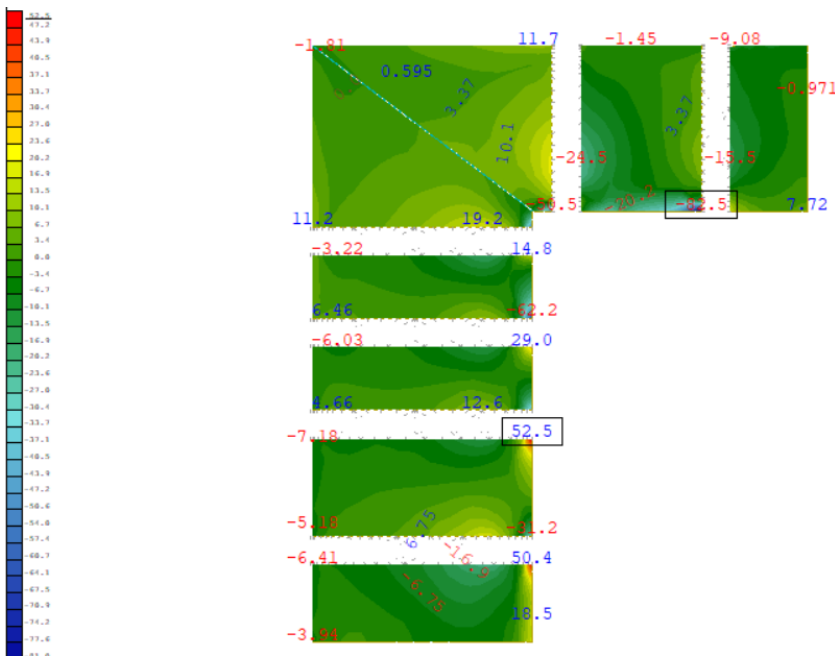


Figure 31 Vertical Shear forces  $V_x$  [kN/m] in the slab for ULS eq. 6.10.b. Shear D-regions have been removed from the plot, according to recommendations from Eurocode. The peak values near the supports are due to the connection with the plate and have been neglected as the design of this detail is not covered here.

The shear forces and bending moments show maximum values over the support beams of around 24.5 kN/m and 12.4 kNm/m (area in compression in the lower half of the cross section) at beam B5. There are other peaks near the supports that are due to the local effects of the plate-beam connection. This area is a D-region and can no longer be analyzed with beam-shell elements, some local reinforcement need to be accounted for near the bolted connections with the support beams.

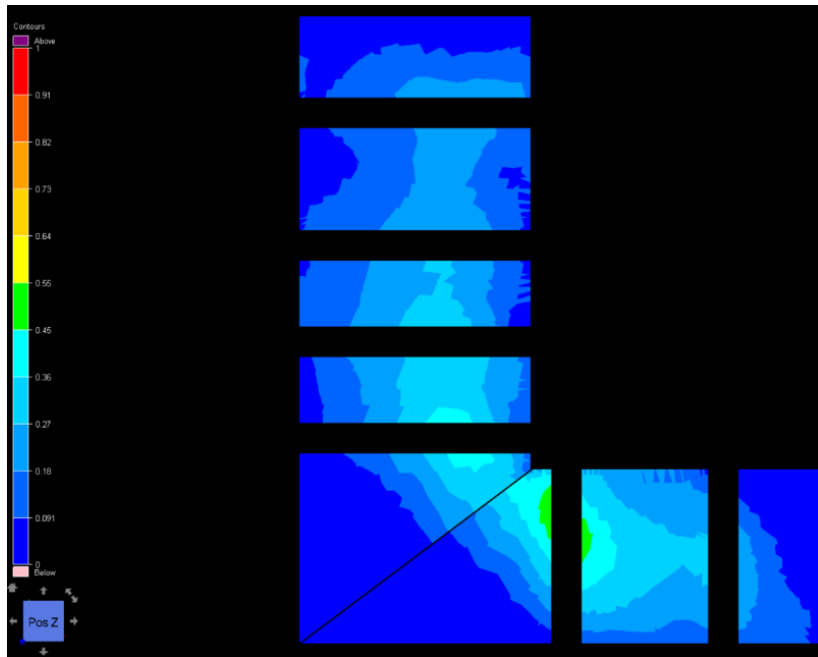


Figure 32 Utilization ratios for concrete compression in bending ( $UR_c$ ).

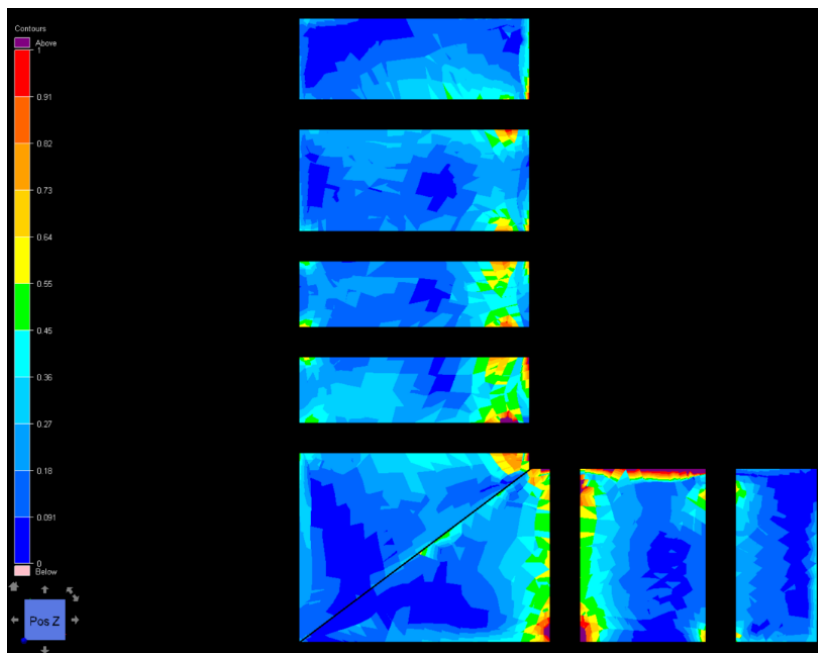


Figure 33 Utilization ratios for concrete compression due to shear ( $UR_{sc}$ ). The values over 1 near the edges are very local and will be dependent on the type of connection between the support beams and the concrete slab. Some local reinforcement needs to be accounted for near those bolted connections.

#### 3.1.1.4 Front beams

In the figures below the design forces for the beams are shown (for the support and front beams). The design of the front beams is further discussed in Section 3.1.1.5.

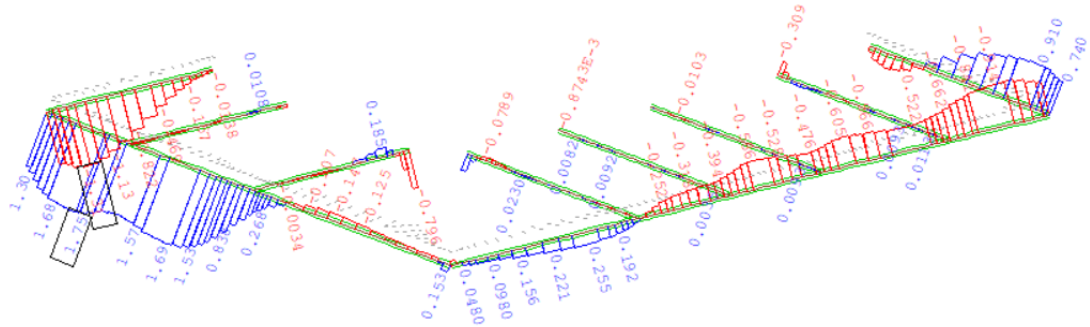


Figure 34 Torsion moment [kNm] in beams.

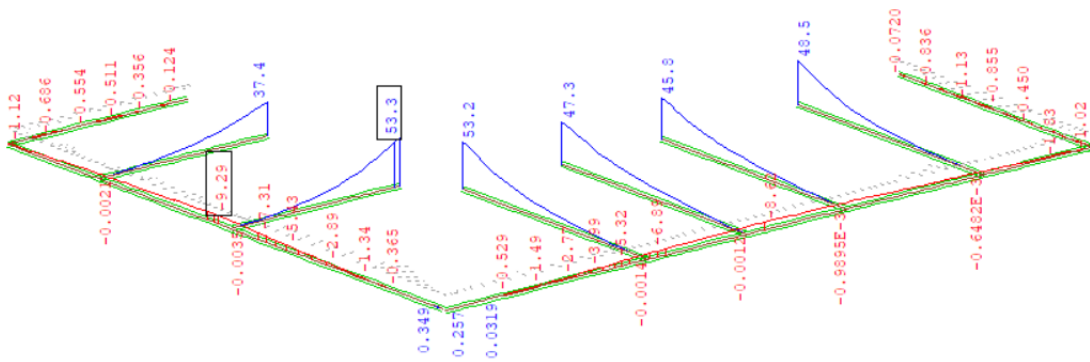


Figure 35 Bending moment [kNm] in beams.

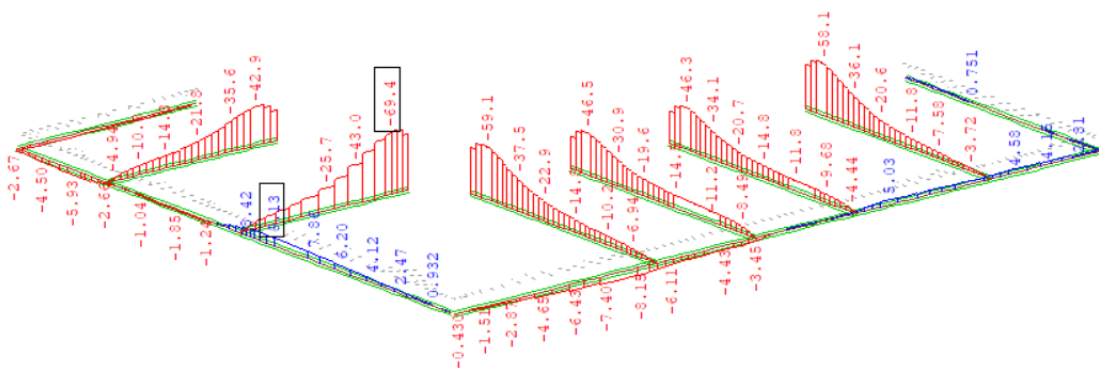


Figure 36 Vertical shear force [kN] in beams.

### 3.1.1.5 Design of front beams

The front beams are an important structural element as they allow a more even distribution of forces between the couplings as well as they stiffen the concrete slab against torsion. These beams can either be made of concrete and casted together with the plate to ensure a full coupling or made of steel (with the same cross section as the support beams, for example) and connected to the slab with

bolts through the concrete. In the latter option the transfer of moments from the slab to the beam might not be full, though.

The front beams are mainly subject to torsion, shear and bending. A beam requires some minimum shear reinforcement according to Eurocode 2 section 6.2.1 paragraph 4. In addition, the torsion requires stirrups, therefore, the cross section of the beam must have enough width to hold stirrups and the longitudinal reinforcement, a minimum of 120 mm is recommended for this purpose. The cross section for the given examples is shown in Figure 37

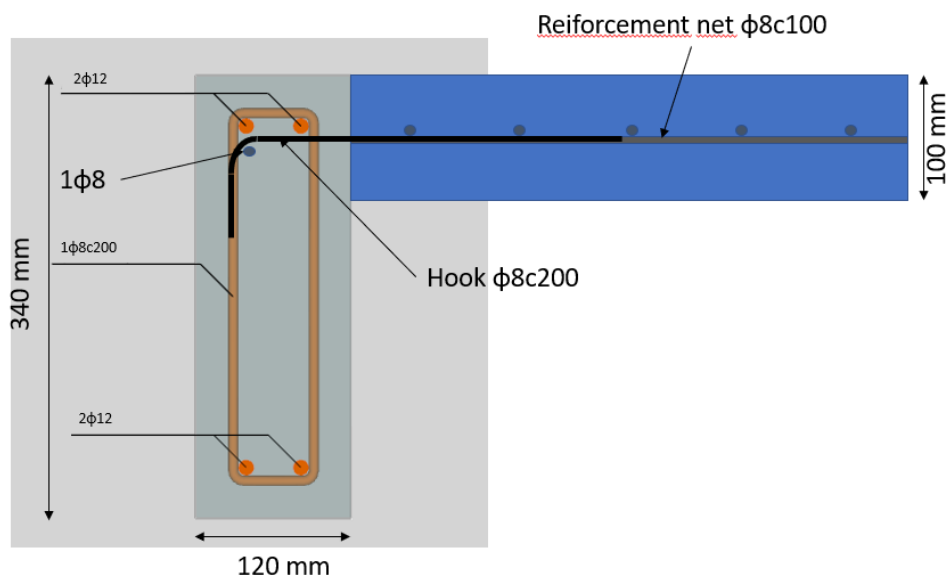


Figure 37 Front beam cross section with reinforcement and connection with the concrete slab

The design of the reinforcement is strongly influenced by the need of stirrups, this requires 4 bars on each corner of the stirrup which gives 25 kNm capacity against bending and 22 kN against shear, more than enough for this beam. The maximum torsion capacity of this solution is around 2 kNm.

The transmission of moments from the balcony slab is possible due to the connection of the reinforcement net and the stirrups of the beam, by means of a φ8 90 ° hook.



### 3.1.2 Skewed beams

The layout of beams and the geometry is shown in the figure below. With this solution we are not utilizing the front beam as a structural member, but the BWC's are skewed to carry the corner of the balcony.

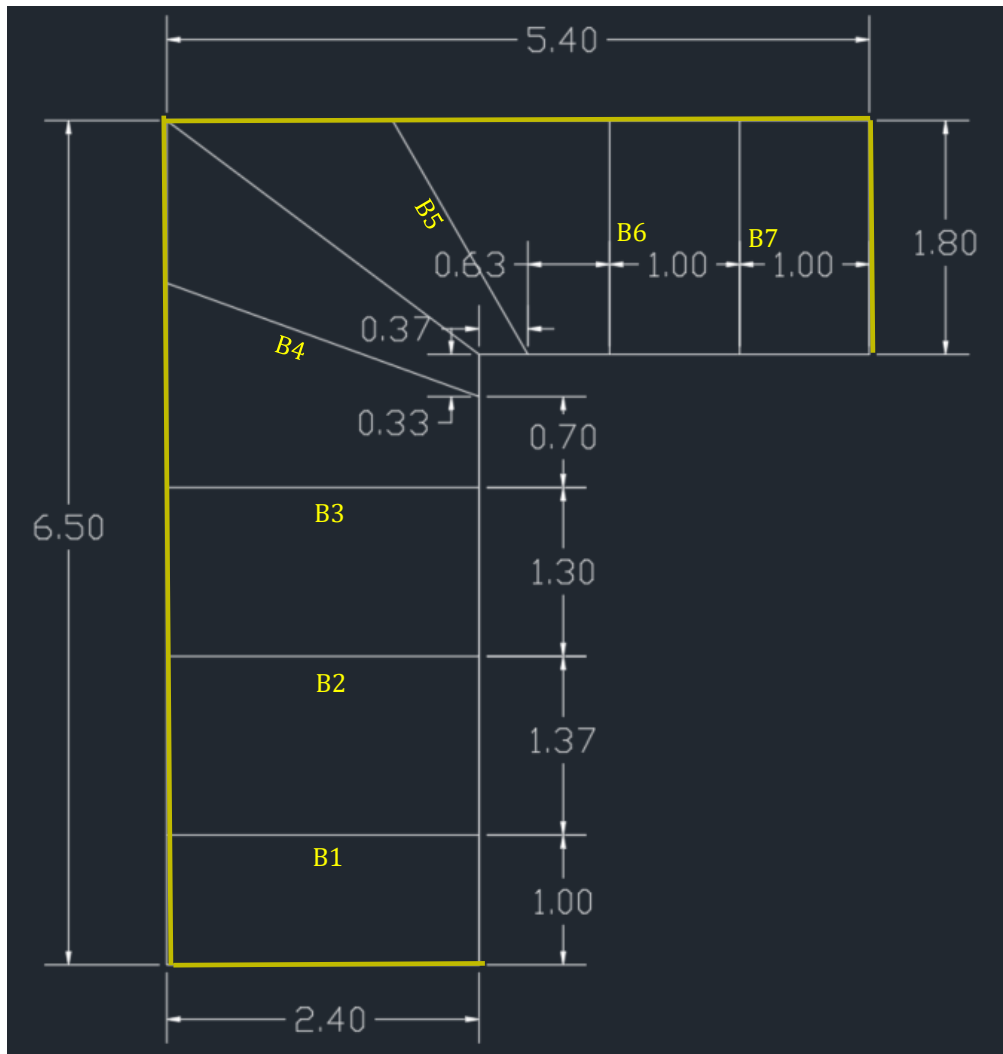


Figure 38 Alternative with 5 straight and 2 skewed beams and a front beam.

The reinforcement of the balcony slab is as described in section 1.1.1.

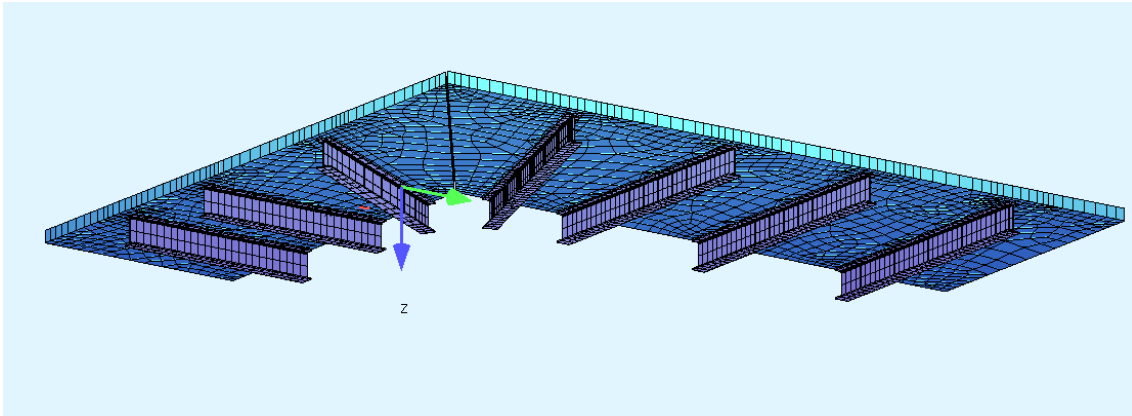


Figure 39 FEA model of the alternative, with beam and shell elements.

### 3.1.2.1 Results

For ULS (eq. 6.10b):

The reactions at the supports (couplings) are as shown in the figure below:

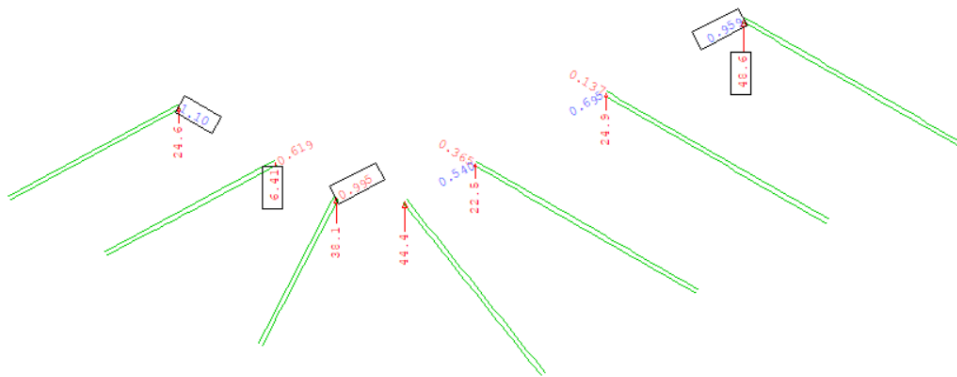


Figure 40 Vertical reaction forces [kN] at the supports for ULS eq. 6.10.b.



### 3.1.2.2 Concrete slab.

The concrete slab is checked against Moment and shear, this is the distribution of maximum moments in x and y directions (x follows the direction of the façade and y goes in the direction of the straight beams).

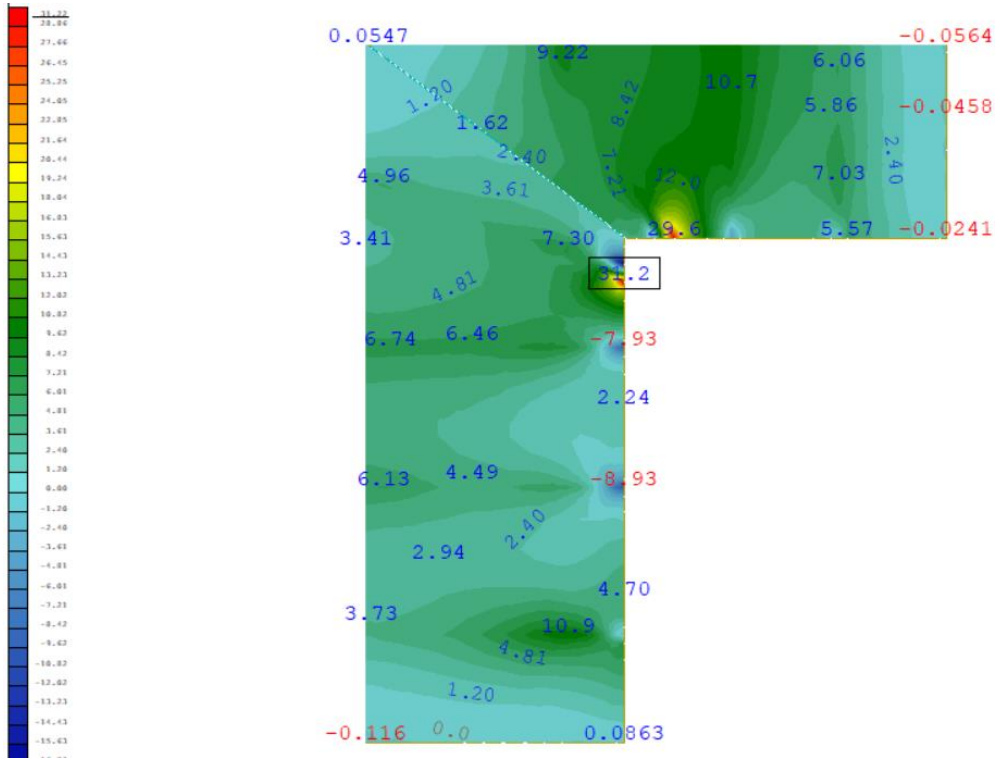


Figure 43 Bending moments  $m_x$  [kNm/m] in the slab for ULS eq. 6.10.b, in local x direction (parallel to the building).

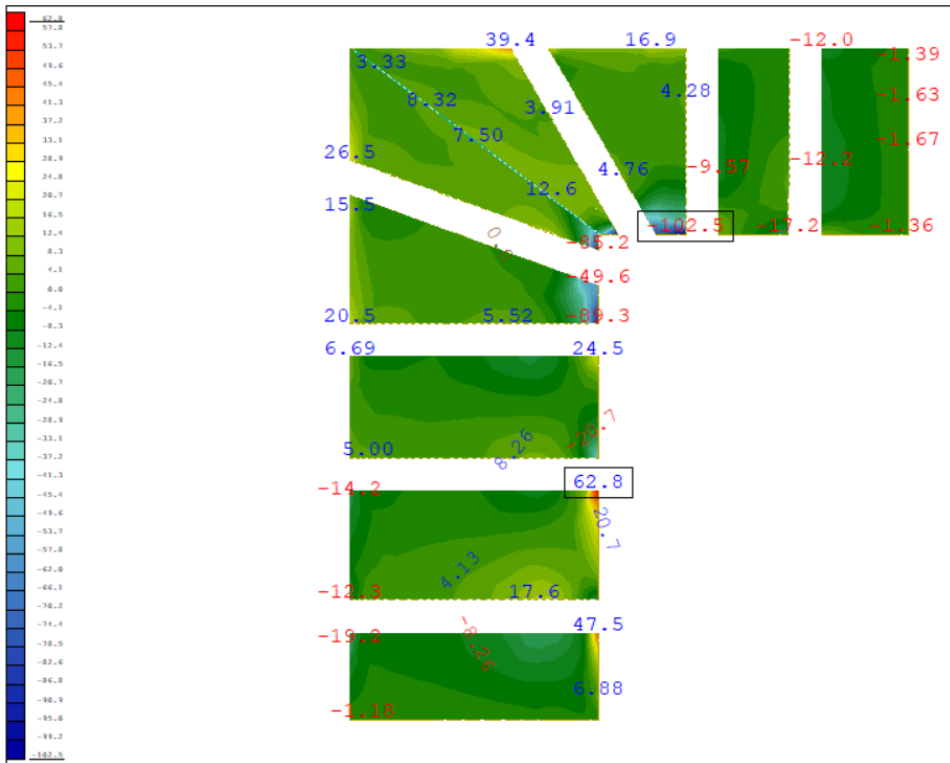


Figure 44 Vertical shear forces  $V_x$  [kN/m] in the slab for ULS eq. 6.10.b. Shear D-regions have been removed from the plot, according to recommendations from Eurocode. The peak values near the supports are due to the connection with the plate and have been neglected, some local reinforcement needs to be placed at those points.

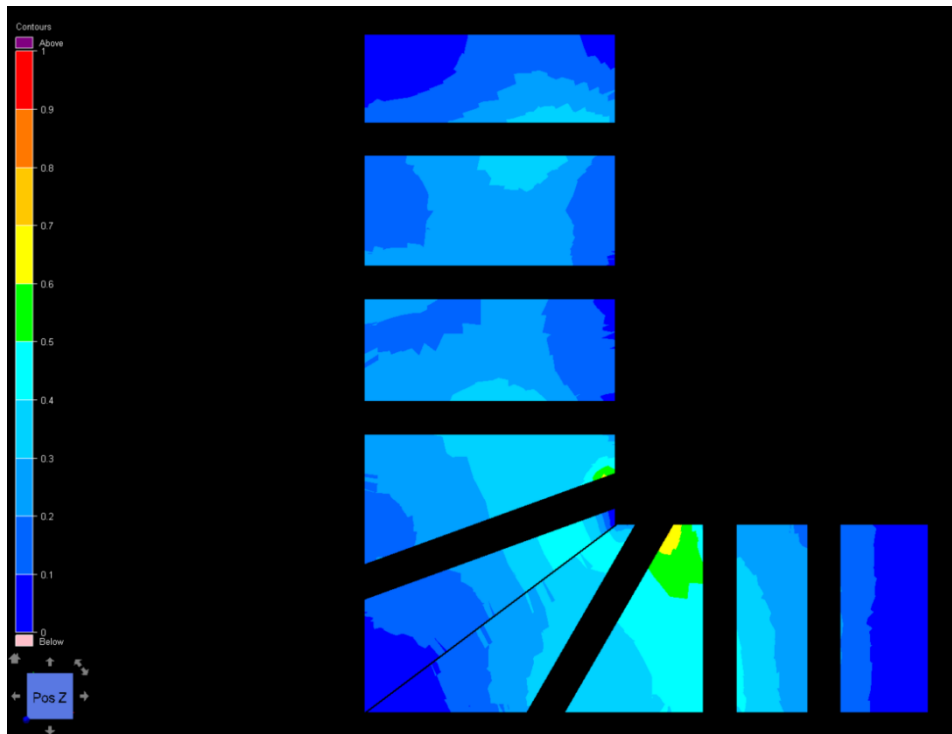


Figure 45 Utilization ratios for concrete compression in bending ( $UR_c$ ).

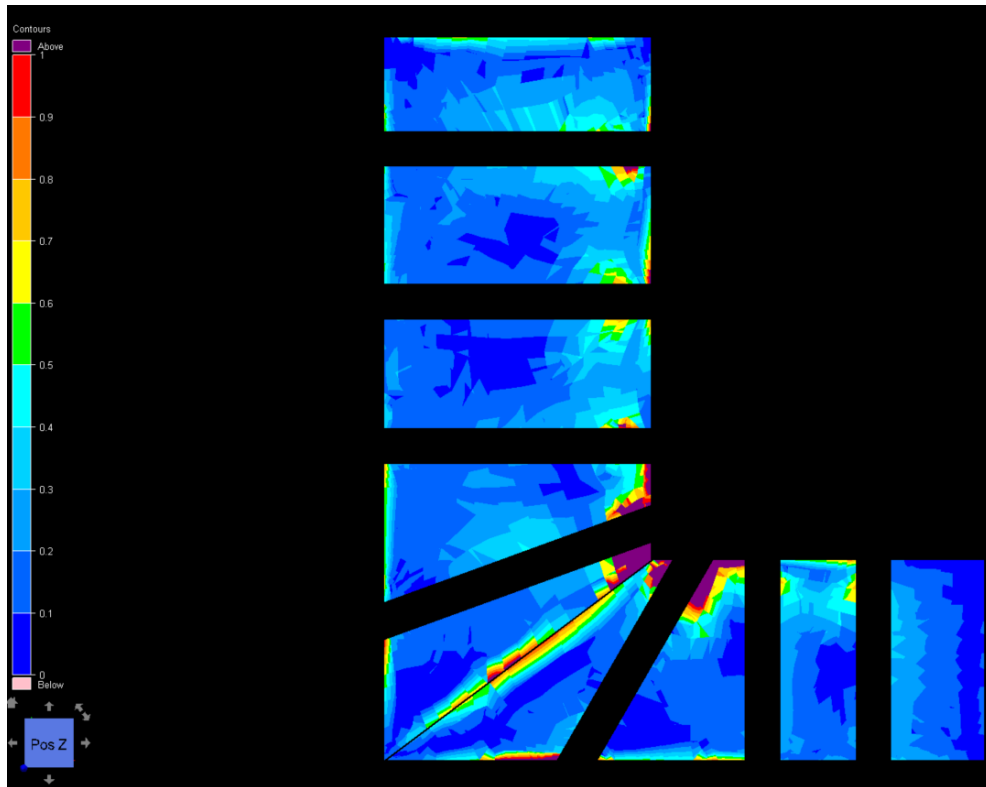


Figure 46 Utilization ratios for concrete compression due to shear ( $UR_{sc}$ ). The values over 1 are due to the local shear concentrations on the skewed beams near the couplings that cannot be analyzed with plate theory.

### 3.1.2.3 Support beams

In the figures below the design forces for the support beams are shown.

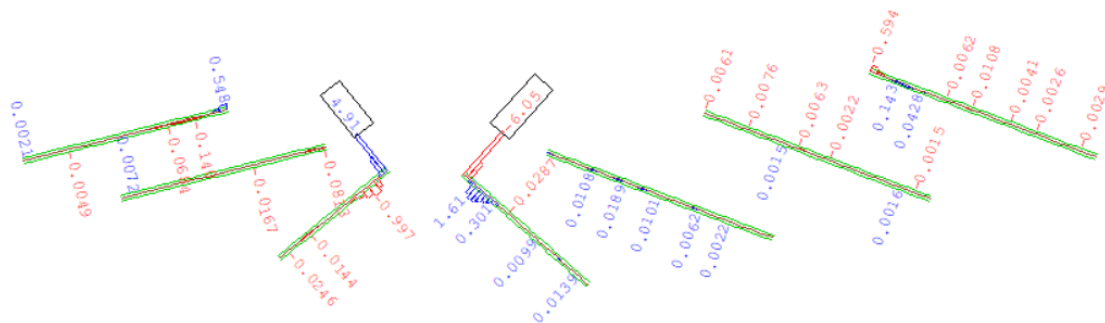


Figure 47 Torsion moment [kNm] in support beams.

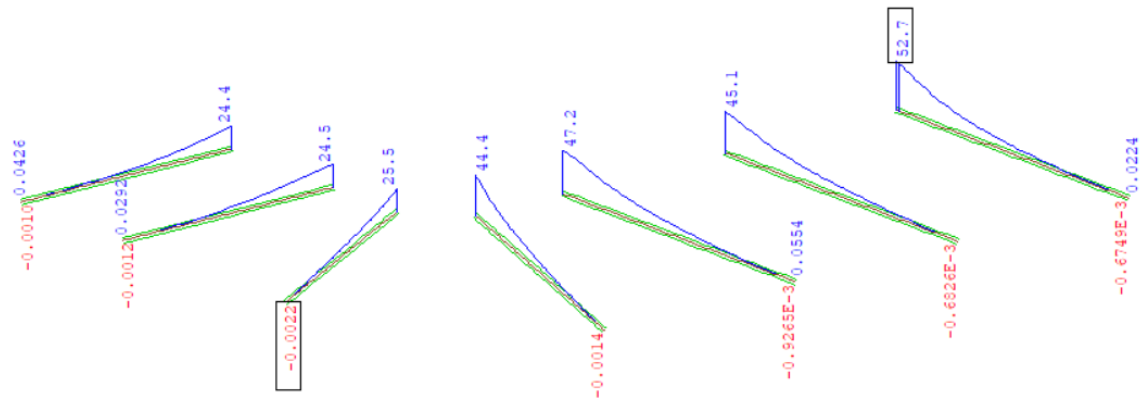


Figure 48 Bending moment [kNm] in support beams.

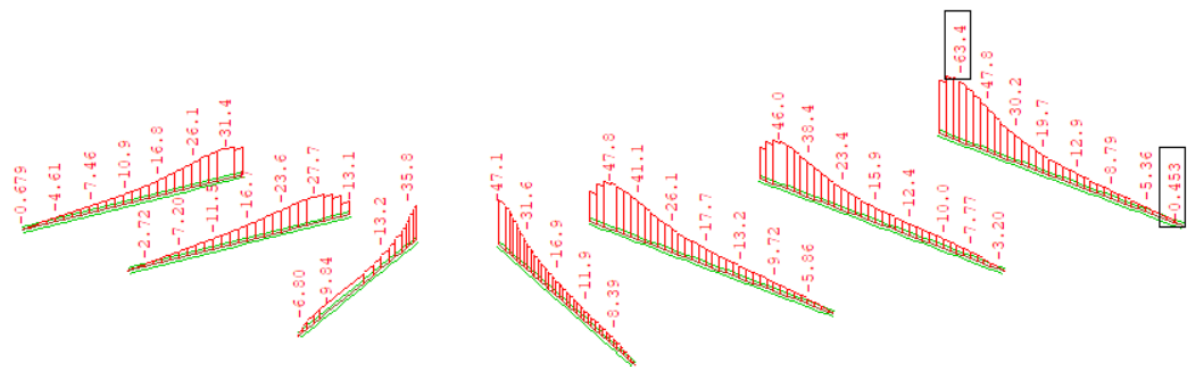


Figure 49 Vertical shear force [kN] in support beams.

## REFERENCES

- [1] Odd Einar Helmersen (Descartes AS), "Memo 750: Veidledning BWC- Utkraget," Invisible Connections AS, 2022.
- [2] "NS-EN 1993:2005+NA 2009 Eurokode: Prosjektering av stålkonstruksjoner - Del 1-8: Knutepunkter og forbindelser," Standard Norge, 2005.
- [3] Odd Einar Helmersen (Descartes AS), "Memo 755: BWC 40 U Modal Analyser," Invisible Connections AS, 2020.
- [4] "NS-EN 1990:2002+A1:2005+NA:2016 Eurokode: Grunnlag for prosjektering av konstruksjoner,," Standard Norge, 2005.



