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DTF / DTS	Siste rev.: 25.05.2016	Sign.:
EXAMPLE CALCULATIONS	Dok. nr.: K4-10/30E	Control: ps
DESIGN		

EXAMPLE CALCULATIONS DTF / DTS

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PART 1 – DTF120 USED IN DT450

1.1 GENERAL

1.1.1 QUALITIES

Concrete C45/55:

$$\begin{aligned}f_{ck} &= 45,0 \text{ MPa} && \text{EC2, Table 3.1} \\f_{cd} &= \alpha_{cc} \times f_{ck} / \gamma_c = 0,85 \times 45 / 1,5 = 25,5 \text{ MPa} && \text{EC2, Clause 3.15} \\f_{ctd} &= \alpha_{ct} \times f_{ctk,0,05} / \gamma_c = 0,85 \times 2,70 / 1,5 = 1,53 \text{ MPa} && \text{EC2, Clause 3.16} \\f_{bd} &= 2,25 \times \eta_1 \times \eta_2 \times f_{ctd} = 2,25 \times 0,7 \times 1,0 \times 1,53 = 2,41 \text{ MPa} && \text{EC2, Clause 8.4.2}\end{aligned}$$

Reinforcement 500C:

$$f_{yd} = f_{yk} / \gamma_s = 500 / 1,15 = 435 \text{ MPa} \quad \text{EC2, Clause 3.2.7}$$

Tendons:

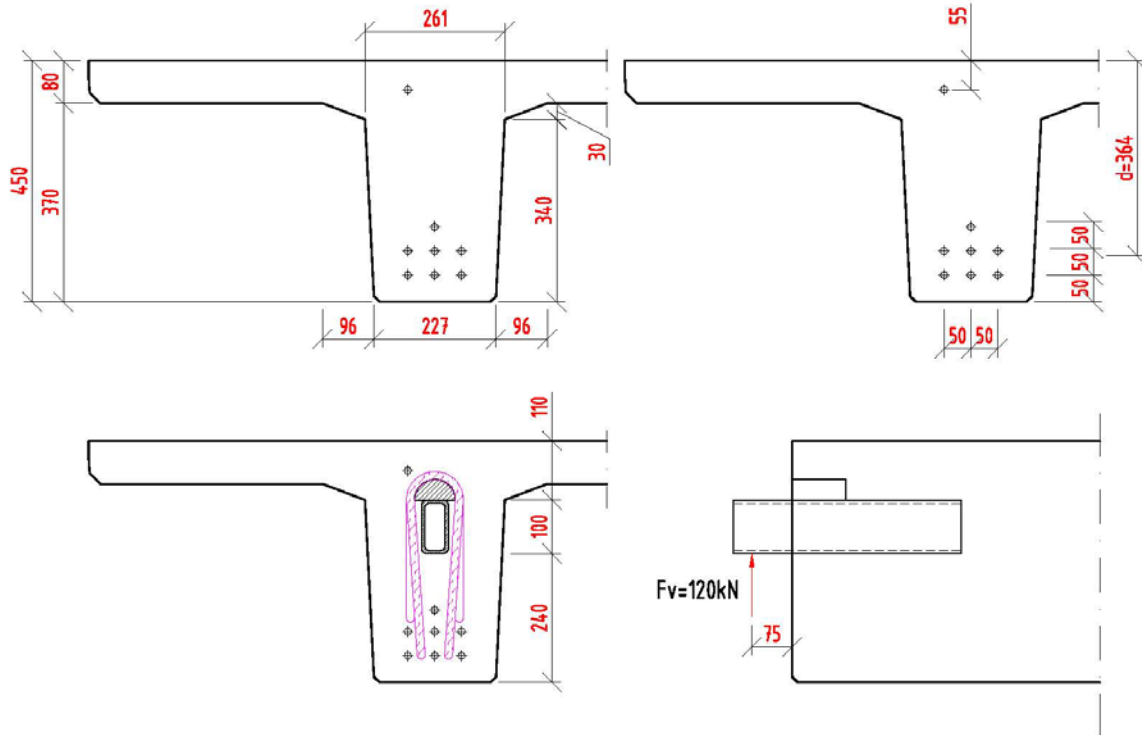
Diameter: $\varnothing=12,7\text{mm}$. (Nominal diameter. Real diameter=11,3mm)

Assumed tension after elastic loss: $P=120\text{kN}$.

1.1.2 LOAD

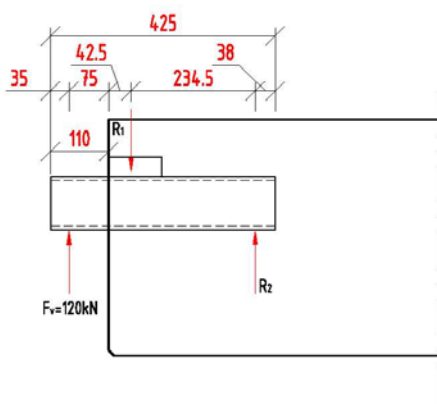
Design load $F_v=120\text{kN}$

1.1.3 GEOMETRY



1.2 CALCULATIONS

1) Equilibrium:



$$R_2 = \frac{F_v \times (75\text{mm} + 42,5\text{mm})}{234,5\text{mm}} = \frac{120\text{kN} \times (75\text{mm} + 42,5\text{mm})}{234,5\text{mm}} \approx 61\text{kN}$$

$$R_1 = F_v + R_2 = 120\text{kN} + 61\text{kN} = 181\text{kN}$$

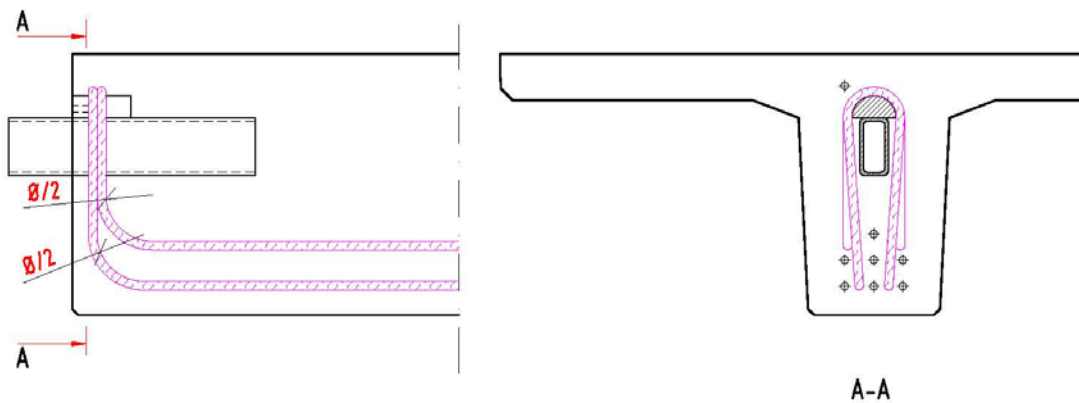
2) Reinforcement:

$$A_{R1} = \frac{181\text{kN}}{435\text{MPa}} = 416\text{mm}^2 \rightarrow 2\phi 12\text{stirrups} = 452\text{mm}^2$$

$$A_{R2} = \frac{61\text{kN}}{435\text{MPa}} \approx 140\text{mm}^2$$

3) Bending of anchoring reinforcement:

Minimum mandrel diameter, $\phi_{m,\min}$:



Allowable concrete stress in node, EC2, clause 6.5.2:

$$f_{cd2} = 0,6 \times \left(1 - \frac{f_{ck}}{250}\right) \times f_{cd}$$

$$= 0,6 \times \left(1 - \frac{45}{250}\right) \times 25,5$$

$$= 12,5\text{MPa}$$

Actual concrete stress in node:

$$\sigma_c = \frac{R_1}{b \times \varnothing_m \times \sin \theta \times \cos \theta}$$

$$b=227\text{mm}$$

\varnothing_m = Mandrel diameter of front reinforcement

θ =assume concrete strut in 45degrees.

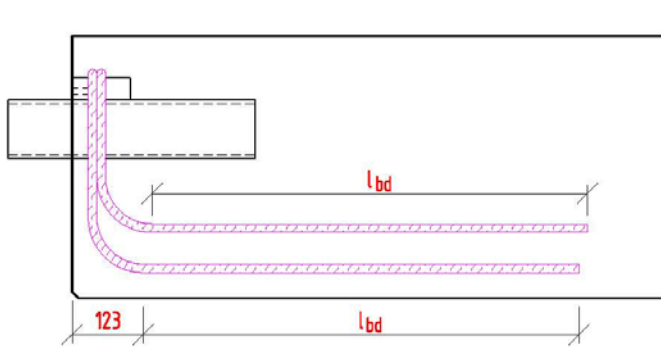
Solving for \varnothing_m :

$$\varnothing_m = \frac{R_1}{b \times \sigma_c \times \sin \theta \times \cos \theta}$$

$$\Rightarrow \varnothing_{m,\min} = \frac{181000N}{227\text{mm} \times 12,5\text{MPa} \times \sin(45) \times \cos(45)} = 128\text{mm}$$

\Rightarrow Select mandrel diameter: $\varnothing_m=160\text{mm}$

4) Anchoring of $\varnothing 12$ stirrups in front, EC2 clause 8.4.3 and 8.4.4:



$$l_{bd} = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_4 \times \alpha_5 \times l_{b,\text{reqd}} \geq l_{b,\text{min}}$$

$$l_{b,\text{reqd}} = \frac{\varnothing}{4} \times \frac{\sigma_{sd}}{f_{bd}}$$

$$\text{Stress in stirrup: } \sigma_{sd} = \frac{181000 / 4}{\pi \times 6^2} = 400\text{MPa}$$

$$l_{b,\text{reqd}} = \frac{12}{4} \times \frac{400}{2,41} = 497\text{mm}$$

$$l_{b,\text{min}} = \max(0,3 \times l_{b,\text{reqd}}; 10 \times \varnothing; 100\text{mm}) = 150\text{mm}$$

Table 8.2: Straight bar:

$$\alpha_1 = 1,0$$

Table 8.2: Concrete cover:

$$\alpha_2 = 1 - 0,15 \times (c_d - 3 \times \phi) / \phi$$

Neglecting any positive effect of concrete cover, selecting $\alpha_2 = 1,0$

Table 8.2: Confinement by reinforcement:

$$\alpha_3 = 1 - k \times \lambda$$

Neglecting any positive effect of transverse reinforcement, selecting $\alpha_3 = 1,0$

Table 8.2: Confinement by welded transverse reinforcement:

$$\alpha_4 = 1,0$$

Not relevant.

Table 8.2: Confinement by transverse pressure:

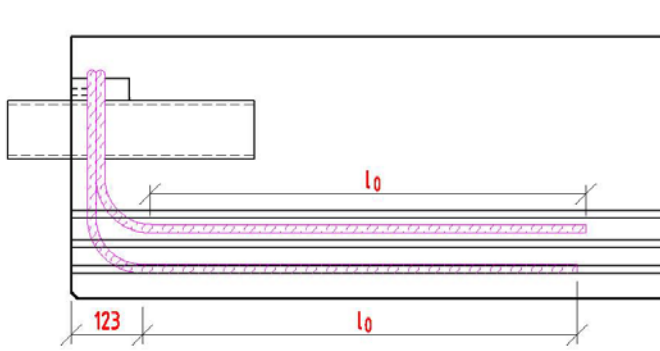
$$\alpha_5 = 1,0$$

Not relevant.

$$\alpha_2 \times \alpha_3 \times \alpha_5 = 1,0 \times 1,0 \times 1,0 = 1,0 > 0,7 - \text{OK}$$

$$l_{bd} = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,0 \times 497 \text{mm} = 497 \text{mm}$$

5) Lap of $\phi 12$ stirrups, EC2 clause 8.7.3:



$$l_0 = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_5 \times \alpha_6 \times l_{b,reqd} \geq l_{0,min}$$

Required lap length, $\phi 12$:

$$l_{b,reqd} = \frac{\phi}{4} \times \frac{\sigma_{sd}}{f_{bd}} = \frac{12}{4} \times \frac{400}{2,41} = 497 \text{mm}$$

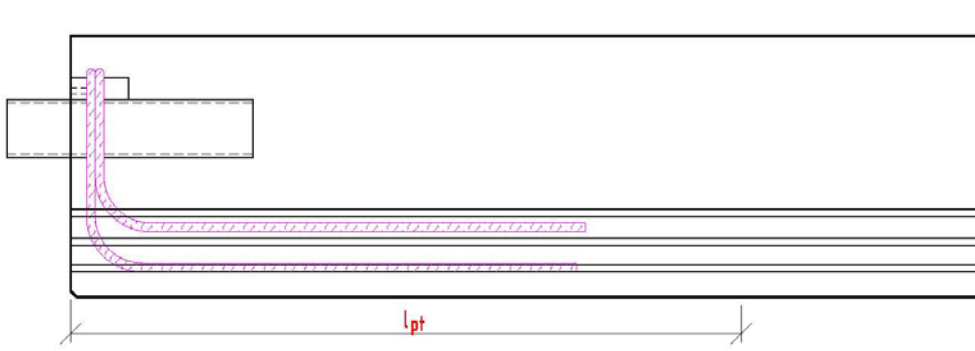
$$l_{0,min} = \max(0,3 \times \alpha_6 \times l_{b,reqd}; 15 \times \phi; 200 \text{mm}) = 224 \text{mm}$$

Table 8.2: $\alpha_1, \alpha_2, \alpha_3, \alpha_5 = 1,0$ as calculated in clause 2).

Table 8.3: $\alpha_6=1.5$ (All reinforcement is lapped)

$$\Rightarrow l_0 = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,5 \times 497 \text{mm} = 746 \text{mm} \Rightarrow \approx 750 \text{mm}$$

6) Transmission length – tendons, EC2 clause 8.10.2.2:



Bond stress:

$$f_{bpt} = \eta_{p1} \times \eta_1 \times f_{ctd}(t)$$

$$\eta_{p1} = 3,2 \text{ (assume 3 or 7-wire tendons)}$$

$$\eta_1 = 1,0 \text{ (assume "good bond conditions")}$$

$$f_{ctd}(t) = \alpha_{ct} \times 0,7 \times f_{ctm}(t) / \gamma_c$$

$$\alpha_{ct} = 0,85$$

$$f_{ctm}(t) = (\beta_{cc}(t))^\alpha \times f_{ctm}$$

$$\beta_{cc}(t) = \exp(s(1-28/t)^{1/2})$$

Assume: Release after $t = 1$ day

Assume: $s = 0,2$

$$\Rightarrow \beta_{cc}(t) = \exp[0,2 \times \{1 - (28/1)^{1/2}\}] = 0,423$$

$$\alpha = 1 \text{ (} t < 28 \text{ days)}$$

$$f_{ctm} = 3,8 \text{MPa}$$

$$\Rightarrow f_{ctm}(t) = 0,423^1 \times 3,8 \text{MPa} = 1,60 \text{MPa}$$

$$f_{ctd}(t) = 0,85 \times 0,7 \times 1,60 \text{MPa} / 1,5 = 0,635 \text{MPa}$$

$$\Rightarrow f_{bpt} = 3,2 \times 1,0 \times 0,635 \text{MPa} = 2,03 \text{MPa}$$

Transmission length:

$$l_{pt} = \alpha_1 \times \alpha_2 \times \varnothing \times \sigma_{pmo} / f_{bpt}$$

$$\alpha_1 = 1,0 \text{ (assume gradual release)}$$

$$\alpha_2 = 0,19 \text{ (assume 3 or 7-wire tendons)}$$

$$\varnothing = 12,7 \text{mm (Nominal diameter of tendon. Real diameter = 11,3mm)}$$

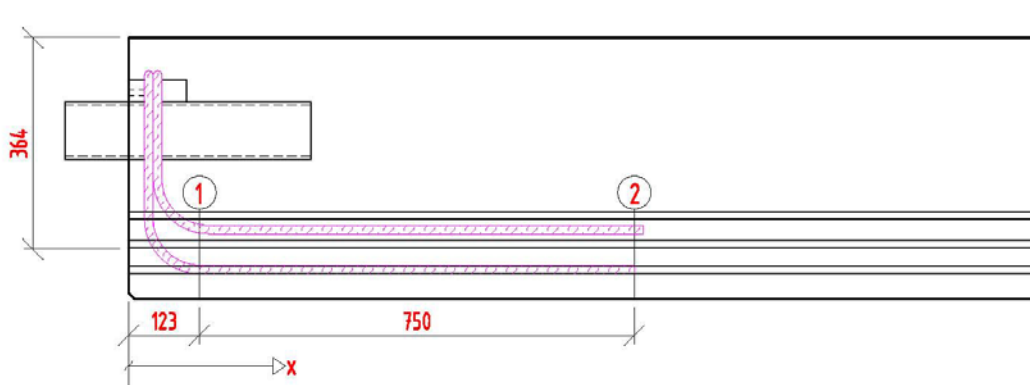
$$\sigma_{pmo} = 1200 \text{MPa}$$

$$\Rightarrow l_{pt} = 1,0 \times 0,19 \times 12,7 \text{mm} \times 1200 \text{MPa} / 2,03 \text{MPa} = 1426 \text{mm}$$

$$\Rightarrow l_{pt1} = 0,8 \times l_{pt} = 0,8 \times 1426 \text{mm} = 1141 \text{mm. To be used in evaluation of local cross section stresses.}$$

$$\Rightarrow l_{pt2} = 1,2 \times l_{pt} = 1,2 \times 1426 \text{mm} = 1711 \text{mm. To be used in evaluation of anchorage. Stress after all losses assumed as } 0,9\sigma_{pmo}. (10\% \text{ loss})$$

7) Anchoring:



Assuming the horizontal part of the front anchoring bar is 750mm (\approx equals the minimum calculated lap length). I.e. the bar ends at $x=123+750=873\text{mm}$.

Section 1:

Force anchored in the tendons (7 tendons) at $x=123\text{mm}$:

Assume 10% loss of pre-stressing force:

$$F_{sp1} = 7 \times P \times 0,9 \times 123 \text{mm} / 1711 \text{mm} = 7 \times 120 \text{kN} \times 0,9 \times 123 \text{mm} / 1711 \text{mm} = 54 \text{kN}$$

Force anchored in $\varnothing 12$:

$$F_{\varnothing 12} = 181 \text{kN}$$

Total anchored force:

$$F = F_{sp1} + F_{\varnothing 12} = 54 \text{kN} + 181 \text{kN} = 235 \text{kN}$$

Tension in reinforcement at $x=123\text{mm}$: (clause 6.2.3(7))

$$S(x) = M(x) / z + 0,5 V_{Ed} \times (\cot(\theta) - \cot(\alpha))$$

$$= M(x) / z + 0,5 \times V_{Ed} \times (\cot(45) - \cot(90)) \text{ (assume 45 degrees concrete struts and vertical links)}$$

$$= M(x) / z + 0,5 \times V_{Ed} \times (1 - 0)$$

$$= M(x) / z + 0,5 \times V_{Ed}$$

Moment at $x=123$:

$$M(x=123) = 120 \text{kN} \times (123 + 75) \text{mm} = 23,8 \text{kNm}$$

Assume: $z = 0,9d = 0,9 \times 364 \text{mm} = 328 \text{mm}$ (approximately)

$$S(x=123) = 23,8 \text{ kNm} / 0,328 \text{ m} + 181 \text{ kN} / 2 = 163 \text{ kN}$$

⇒ The anchoring at $x=123 \text{ mm}$ is sufficient.

Section 2:

Force anchored in the tendons (7 tendons) at $x=873 \text{ mm}$:

Assume 10% loss of pre-stressing force:

$$F_{sp1} = 7 \times P \times 0,9 \times 873 \text{ mm} / 1711 \text{ mm} = 7 \times 120 \text{ kN} \times 0,9 \times 873 \text{ mm} / 1711 \text{ mm} = 386 \text{ kN}$$

Force anchored in $\varnothing 12$:

$$F_{\varnothing 12} = 0 \text{ kN}$$

Total anchored force:

$$F = F_{sp1} + F_{\varnothing 12} = 386 \text{ kN} + 0 \text{ kN} = 386 \text{ kN}$$

Tension in reinforcement at $x=873 \text{ mm}$: (clause 6.2.3(7))

$$S(x) = M(x) / z + 0,5 V_{Ed} \times (\cot(\theta) - \cot(\alpha))$$

$$= M(x) / z + 0,5 \times V_{Ed} \times (\cot(45) - \cot(90)) \text{ (assume 45 degrees concrete struts and vertical links)}$$

$$= M(x) / z + 0,5 \times V_{Ed} \times (1 - 0)$$

$$= M(x) / z + 0,5 \times V_{Ed}$$

Moment at $x=873$:

$$M(x=873) = 120 \text{ kN} \times (873 + 75) \text{ mm} = 113,8 \text{ kNm}$$

Assume: $z=0,9d=0,9 \times 364 \text{ mm} = 328 \text{ mm}$

$$S(x=873) = 113,8 \text{ kNm} / 0,328 \text{ m} + 120 \text{ kN} / 2 = 407 \text{ kN}$$

⇒ The tendons will not have sufficient anchorage at $x=873 \text{ mm}$.

Calculating the point where the tendons are sufficient anchored to carry the load:

Tension in reinforcement at x :

$$S(x) = 120 \text{ kN} \times (x + 75 \text{ mm}) / 328 \text{ mm} + 120 \text{ kN} / 2$$

Force anchored in the tendons at x (if $x < 1711 \text{ mm}$):

$$F(x) = 7 \times 120 \text{ kN} \times 0,9 \times x / 1711 \text{ mm}$$

Hence:

$$120 \text{ kN} \times (x + 75 \text{ mm}) / 328 \text{ mm} + 120 \text{ kN} / 2 = 756 \text{ kN} \times x / 1711$$

$$0,366x + 87,4 = 0,442x$$

$$0,076x = 87,4$$

$$x = 87,4 / 0,076$$

$$x = 1150 \text{ mm}$$

⇒ The horizontal part of the $\varnothing 12$ anchoring bars has to be extended beyond the calculated required lap length (evaluated in clause 5), and end at $x \geq 1150$.

8) Reinforcement due to splitting stress:

$$A_s = 0,22 \times P_{u1} / f_s$$

$$\Rightarrow A_s = 0,22 \times 7 \times 120000 \text{ N} / 300 \text{ MPa} \\ = 616 \text{ mm}^2$$

To be located within: $0,5 \times (l_{pt1} + h_1) = 0,5 \times (1141 \text{ mm} + 450 \text{ mm}) = 796 \text{ mm} \leq h_1 = 450 \text{ mm} \Rightarrow 450 \text{ mm}$

Corresponds to: $616 \text{ mm}^2 / 0,450 \text{ m} = 1369 \text{ mm}^2 / \text{m}$

9) Links:

a) Required shear reinforcement $x < 277$:

$$\frac{A_s}{s} = \frac{V_{Ed}}{z \times f_{ywd} \times \cot \theta} = \frac{R_1}{z \times f_{ywd} \times \cot \theta} = \frac{181000 \text{ N}}{0,328 \text{ m} \times 435 \text{ MPa} \times \cot(45)} = 1269 \text{ mm}^2 / \text{m}$$

Shear compression:

$$V_{Rd} = \frac{\alpha_{cw} \times b_w \times z \times v_1 \times f_{cd}}{(\cot \theta + \tan \theta)}$$

$\alpha_{cw} = 1,0$ (neglecting effect of pre-tensioning)

$b_w = 227 \text{ mm} - 50 \text{ mm} = 177 \text{ mm}$

$z = 328 \text{ mm}$

$v_1 = 0,6 \times (1 - f_{ck} / 250) = 0,6 \times (1 - 45 / 250) = 0,492$

$\theta =$ assume 45 degrees

$$V_{Rd} = \frac{1,0 \times 177 \text{ mm} \times 328 \text{ mm} \times 0,492 \times 25,5 \text{ MPa}}{1 + 1} = 364,2 \text{ kN} > R_1 \rightarrow OK$$

b) Required shear reinforcement $277 < x < 606$:

$$\frac{A_s}{s} = \frac{V_{Ed}}{z \times f_{ywd} \times \cot \theta} = \frac{F_V}{z \times f_{ywd} \times \cot \theta} = \frac{120000 \text{ N}}{0,328 \text{ m} \times 435 \text{ MPa} \times \cot(45)} = 841 \text{ mm}^2 / \text{m}$$

Shear compression:

$$V_{Rd} = \frac{\alpha_{cw} \times b_w \times z \times v_1 \times f_{cd}}{(\cot \theta + \tan \theta)}$$

$\alpha_{cw} = 1,0$ (neglecting effect of pre-tensioning)

$b_w = 227 \text{ mm}$

$z = 328 \text{ mm}$

$v_1 = 0,6 \times (1 - f_{ck} / 250) = 0,6 \times (1 - 45 / 250) = 0,492$

$\theta =$ assume 45degrees

$$V_{Rd} = \frac{1,0 \times 227mm \times 328mm \times 0,492 \times 25,5MPa}{1+1} = 467kN > F_v \rightarrow OK$$

Summary – reinforcement in end of the DT:

$X < \approx 450mm$: Links due to splitting stresses (8)

$$\frac{A_s}{s} = 1369mm^2 / m$$

Select $\emptyset 8$ stirrups c/c70 (lapped with u-shaped $\emptyset 12$ bars)

$$\frac{A_s}{s} = \frac{\pi \times (4mm)^2 \times 2}{0,07m} = 1436mm^2 / m$$

$\approx 450 < X < \approx 606mm$: Links due to shear force (9).

$$\frac{A_s}{s} = 841mm^2 / m$$

Select $\emptyset 8$ stirrups c/c100

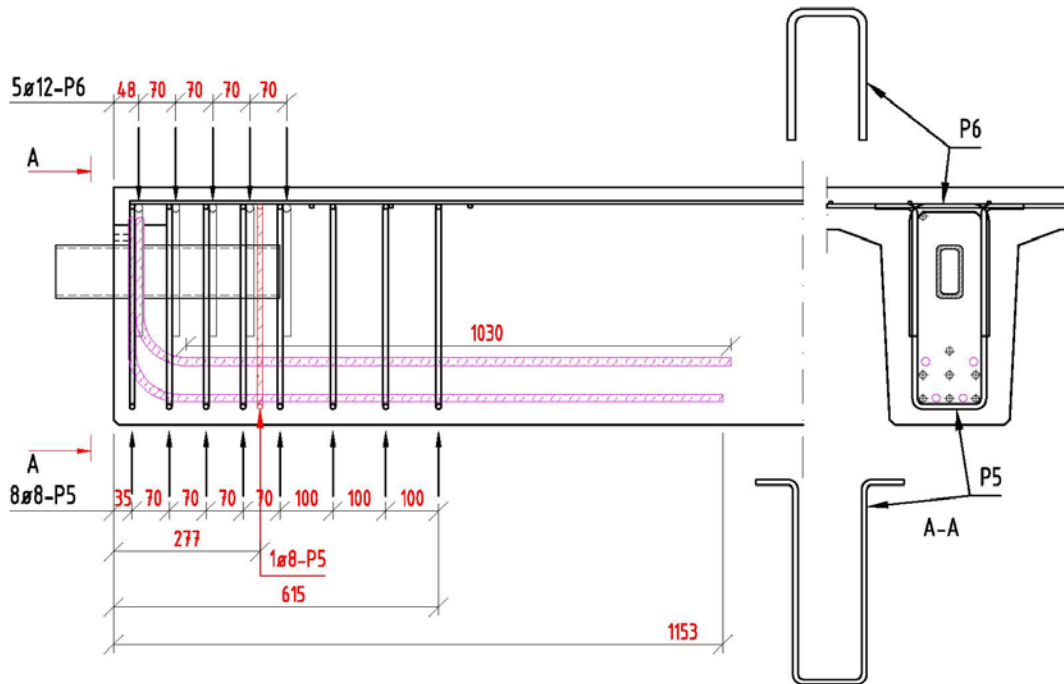
$$\frac{A_s}{s} = \frac{\pi \times (4mm)^2 \times 2}{0,1m} = 1005mm^2 / m$$

$606 < X$: Required shear reinforcement to be evaluated according to shear force distribution in the DT.

At $x=277mm$:

Required reinforcement: $140mm^2 \Rightarrow 1$ extra $\emptyset 8$ link ($100mm^2$) at this location. The remaining $40mm^2$ can be carried by the specified distributed links.

Suggested reinforcement in end of the DT:



PART 2 – DTF200 USED IN A HIGH DT

2.1 GENERAL

2.1.1 QUALITIES

Concrete C45/55:

$$f_{ck} = 45,0 \text{ MPa} \quad \text{EC2, Table 3.1}$$

$$f_{cd} = \alpha_{cc} \times f_{ck} / \gamma_c = 0,85 \times 45 / 1,5 = 25,5 \text{ MPa} \quad \text{EC2, Clause 3.15}$$

$$f_{ctd} = \alpha_{ct} \times f_{ctk,0,05} / \gamma_c = 0,85 \times 2,70 / 1,5 = 1,53 \text{ MPa} \quad \text{EC2, Clause 3.16}$$

$$f_{bd} = 2,25 \times \eta_1 \times \eta_2 \times f_{ctd} = 2,25 \times 0,7 \times 1,0 \times 1,53 = 2,41 \text{ MPa} \quad \text{EC2, Clause 8.4.2}$$

Reinforcement B500C:

$$f_{yd} = f_{yk} / \gamma_s = 500 / 1,15 = 435 \text{ MPa} \quad \text{EC2, Clause 3.2.7}$$

Tendons:

Diameter: $\varnothing=12,7\text{mm}$. (Nominal diameter. Real diameter=11,3mm)

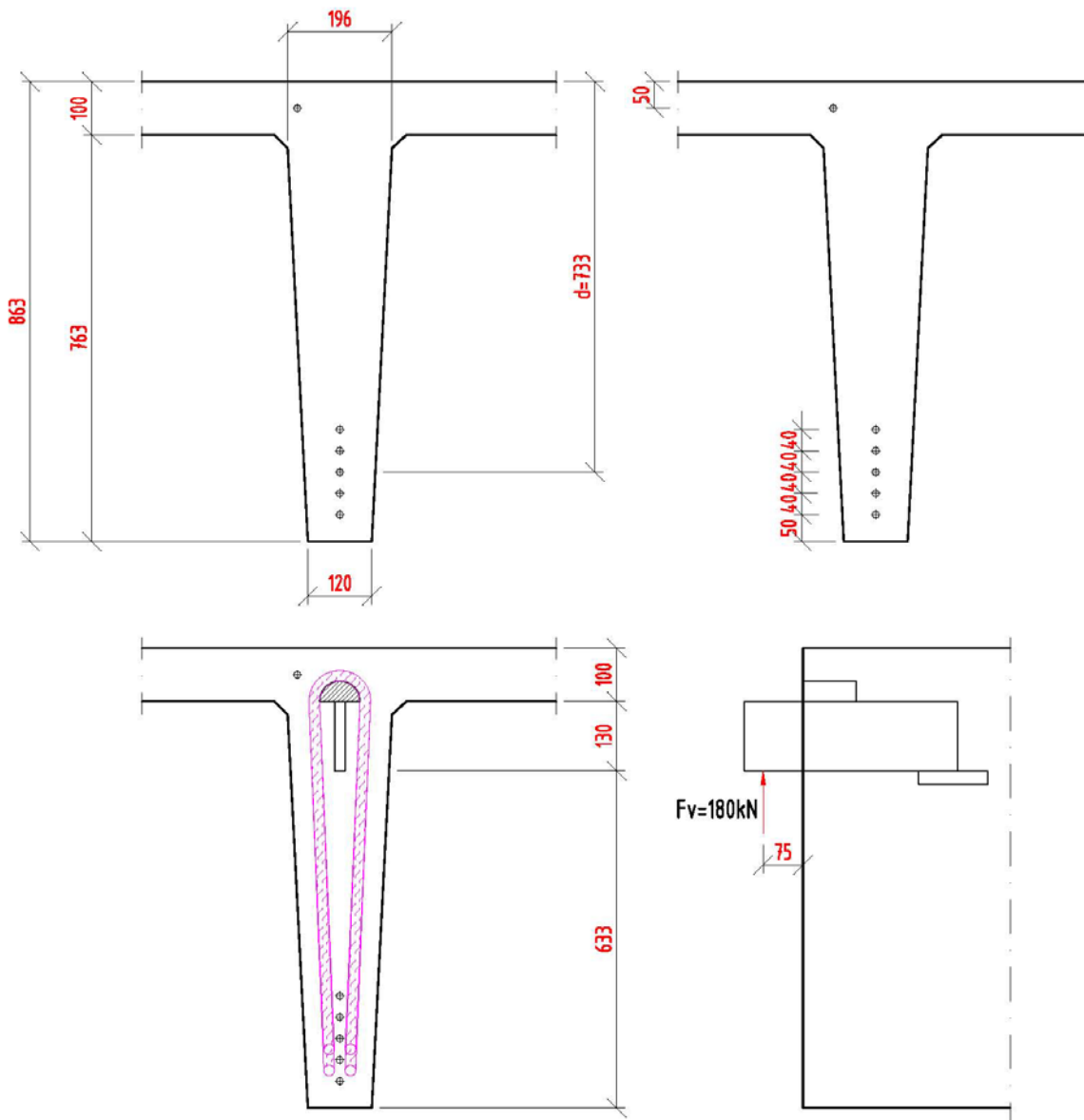
Assumed tension after elastic loss: $P=120\text{kN}$.

2.1.2 LOAD

Design load $F_v=180\text{kN}$

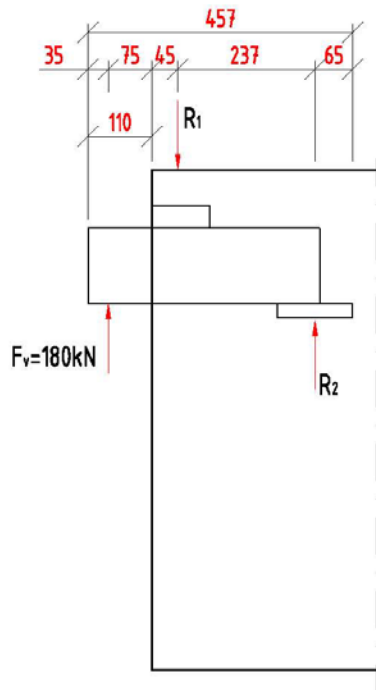
(NB: The design load in the example is less than the ultimate limit load of the unit.)

2.1.3 GEOMETRY



2.2 CALCULATIONS

1) Equilibrium:



$$R_2 = \frac{F_v \times (75\text{mm} + 45\text{mm})}{237\text{mm}} = \frac{180\text{kN} \times (75\text{mm} + 45\text{mm})}{237\text{mm}} \approx 91\text{kN}$$

$$R_1 = F_v + R_2 = 180\text{kN} + 91\text{kN} = 271\text{kN}$$

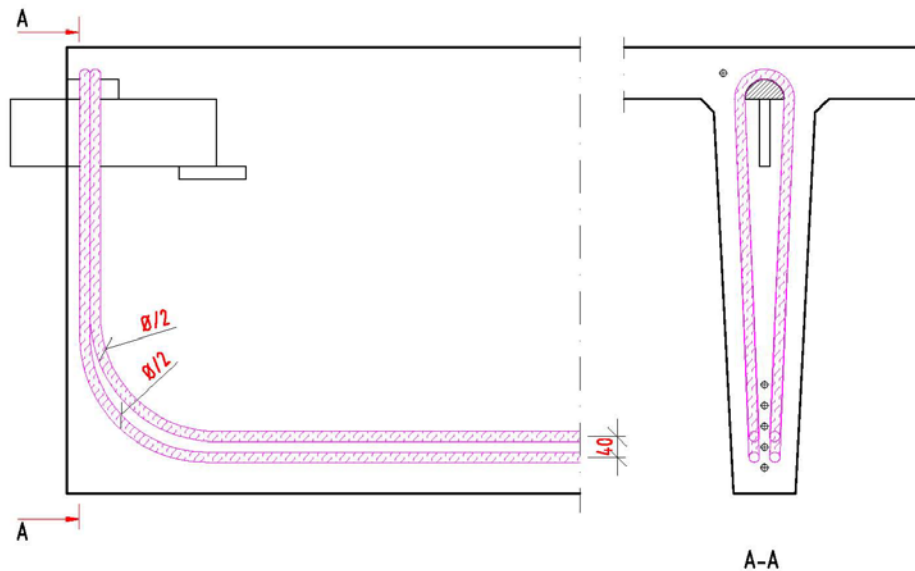
2) Reinforcement:

$$A_{R1} = \frac{271\text{kN}}{435\text{MPa}} = 623\text{mm}^2 \rightarrow 2\phi 16\text{stirrups} = 804\text{mm}^2$$

$$A_{R2} = \frac{91\text{kN}}{435\text{mm}} \approx 210\text{mm}^2$$

3) Bending of anchoring reinforcement:

Minimum mandrel diameter, $\varnothing_{m,min}$:



Allowable concrete stress in node, EC2, clause 6.5.2:

$$\begin{aligned}
 f_{cd2} &= 0,6 \times \left(1 - \frac{f_{ck}}{250}\right) \times f_{cd} \\
 &= 0,6 \times \left(1 - \frac{45}{250}\right) \times 25,5 \\
 &= 12,5 \text{ MPa}
 \end{aligned}$$

Actual concrete stress in node:

$$\sigma_c = \frac{R_1}{b \times \varnothing_m \times \sin \theta \times \cos \theta}$$

$$b = 120 \text{ mm}$$

\varnothing_m = Mandrel diameter of front reinforcement

θ = assume concrete strut in 45 degrees.

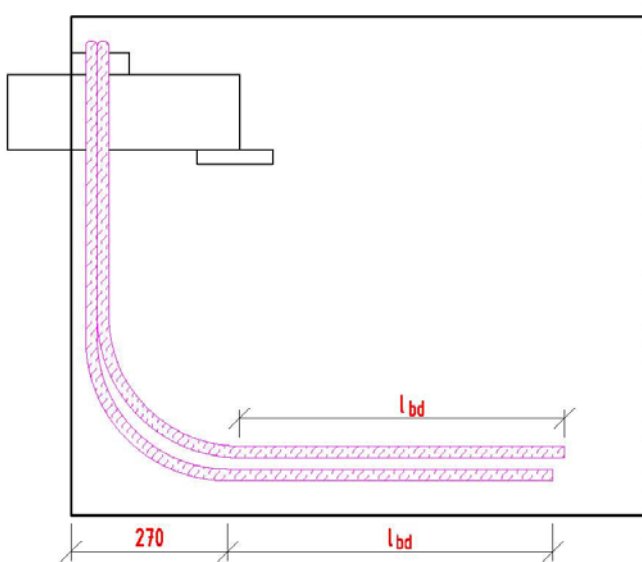
Solving for \varnothing_m :

$$\varnothing_m = \frac{R_1}{b \times \sigma_c \times \sin \theta \times \cos \theta}$$

$$\Rightarrow \varnothing_{m,min} = \frac{271000 \text{ N}}{120 \text{ mm} \times 12,5 \text{ MPa} \times \sin(45) \times \cos(45)} = 361 \text{ mm}$$

\Rightarrow Select mandrel diameter: $\varnothing_m = 450 \text{ mm}$

4) Anchoring of Ø16 stirrups in front, EC2 clause 8.4.3 and 8.4.4:



$$l_{bd} = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_4 \times \alpha_5 \times l_{b,reqd} \geq l_{b,min}$$

$$l_{b,reqd} = \frac{\phi}{4} \times \frac{\sigma_{sd}}{f_{bd}}$$

$$\text{Stress in stirrup: } \sigma_{sd} = \frac{271000 / 4}{\pi \times 8^2} = 337 \text{ MPa}$$

$$l_{b,reqd} = \frac{16}{4} \times \frac{337}{2,41} = 560 \text{ mm}$$

$$l_{b,min} = \max(0,3 \times l_{b,reqd}; 10 \times \phi; 100 \text{ mm}) = 168 \text{ mm}$$

Table 8.2: Straight bar:

$$\alpha_1 = 1,0$$

Table 8.2: Concrete cover:

$$\alpha_2 = 1 - 0,15 \times (c_d - 3 \times \phi) / \phi$$

Neglecting any positive effect of concrete cover, selecting $\alpha_2 = 1,0$

Table 8.2: Confinement by reinforcement:

$$\alpha_3 = 1 - K \times \lambda$$

Neglecting any positive effect of transverse reinforcement, selecting $\alpha_3 = 1,0$

Table 8.2: Confinement by welded transverse reinforcement:

$$\alpha_4 = 1,0$$

Not relevant.

Table 8.2: Confinement by transverse pressure:

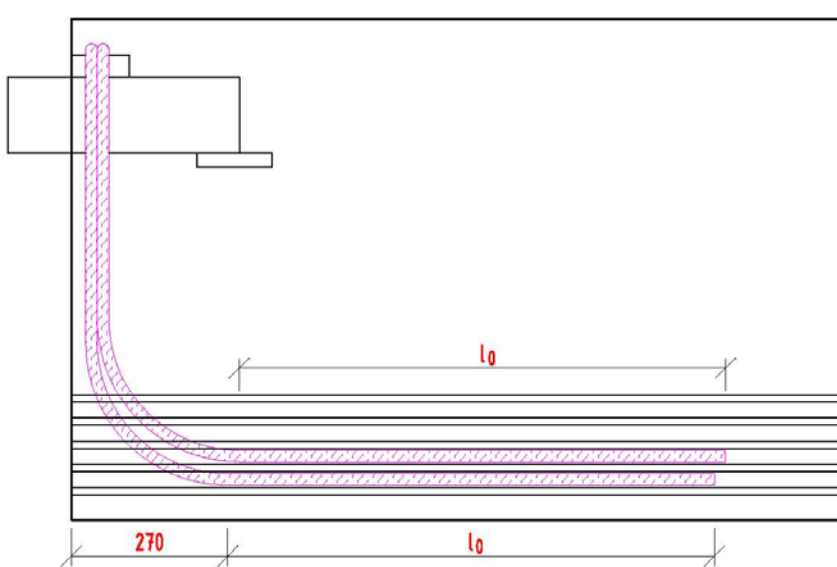
$$\alpha_5 = 1,0$$

Not relevant.

$$\alpha_2 \times \alpha_3 \times \alpha_5 = 1,0 \times 1,0 \times 1,0 = 1,0 > 0,7 - \text{OK}$$

$$l_{bd} = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 560 \text{mm} = 560 \text{mm}$$

5) Lap of Ø16 stirrups, EC2 clause 8.7.3:



$$l_0 = \alpha_1 \times \alpha_2 \times \alpha_3 \times \alpha_5 \times \alpha_6 \times l_{b, \text{reqd}} \geq l_{0, \text{min}}$$

Required lap length, Ø16:

$$l_{b, \text{reqd}} = \frac{\sigma}{4} \times \frac{\sigma_{sd}}{f_{bd}} = \frac{16}{4} \times \frac{337}{2,41} = 560 \text{mm}$$

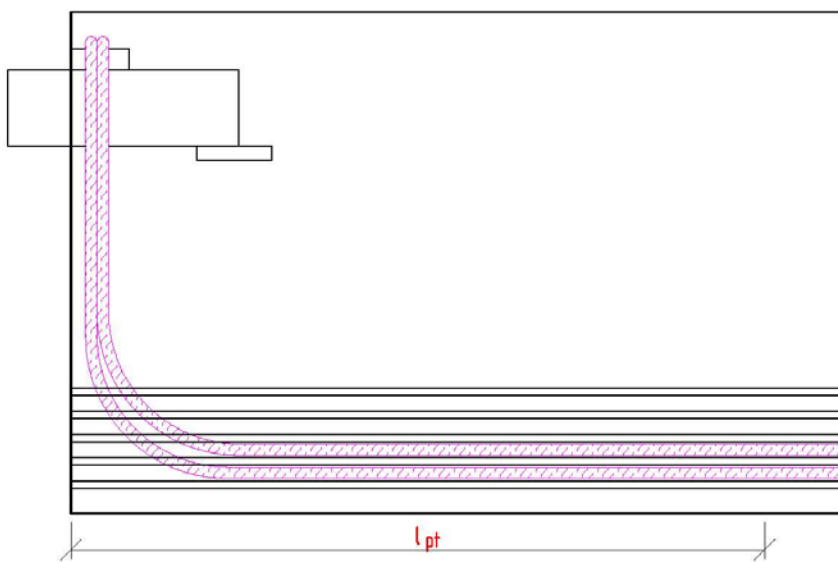
$$l_{0, \text{min}} = \max(0,3 \times \alpha_6 \times l_{b, \text{reqd}}; 15 \times \varnothing; 200 \text{mm}) = 251 \text{mm}$$

Table 8.2: $\alpha_1, \alpha_2, \alpha_3, \text{og } \alpha_5 = 1,0$ as calculated in clause 2).

Table 8.3: $\alpha_6 = 1.5$ (All reinforcement is lapped)

$$\Rightarrow l_0 = 1,0 \times 1,0 \times 1,0 \times 1,0 \times 1,5 \times 560 \text{mm} = 840 \text{mm}$$

6) Transmission length – tendons, EC2 clause 8.10.2.2:



Bond stress:

$$f_{bpt} = \eta_{p1} \times \eta_1 \times f_{ctd}(t)$$

$$\eta_{p1} = 3,2 \text{ (assume 3 or 7-wire tendons)}$$

$$\eta_1 = 1,0 \text{ (assume "good bond conditions")}$$

$$f_{ctd}(t) = \alpha_{ct} \times 0,7 \times f_{ctm}(t) / \gamma_c$$

$$\alpha_{ct} = 0,85$$

$$f_{ctm}(t) = (\beta_{cc}(t))^\alpha \times f_{ctm}$$

$$\beta_{cc}(t) = \exp(s(1-28/t)^{1/2})$$

Assume: Release after $t = 1$ day

Assume: $s = 0,2$

$$\Rightarrow \beta_{cc}(t) = \exp[0,2 \times \{1 - (28/1)^{1/2}\}] = 0,423$$

$$\alpha = 1 \text{ (} t < 28 \text{ days)}$$

$$f_{ctm} = 3,8 \text{ MPa}$$

$$\Rightarrow f_{ctm}(t) = 0,423^1 \times 3,8 \text{ MPa} = 1,60 \text{ MPa}$$

$$f_{ctd}(t) = 0,85 \times 0,7 \times 1,60 \text{ MPa} / 1,5 = 0,635 \text{ MPa}$$

$$\Rightarrow f_{bpt} = 3,2 \times 1,0 \times 0,635 \text{ MPa} = 2,03 \text{ MPa}$$

Transmission length:

$$l_{pt} = \alpha_1 \times \alpha_2 \times \phi \times \sigma_{pmo} / f_{bpt}$$

$$\alpha_1 = 1,0 \text{ (assume gradual release)}$$

$$\alpha_2 = 0,19 \text{ (assume 3 or 7-wire tendons)}$$

$$\phi = 12,7 \text{ mm (Nominal diameter of tendon. Real diameter} = 11,3 \text{ mm)}$$

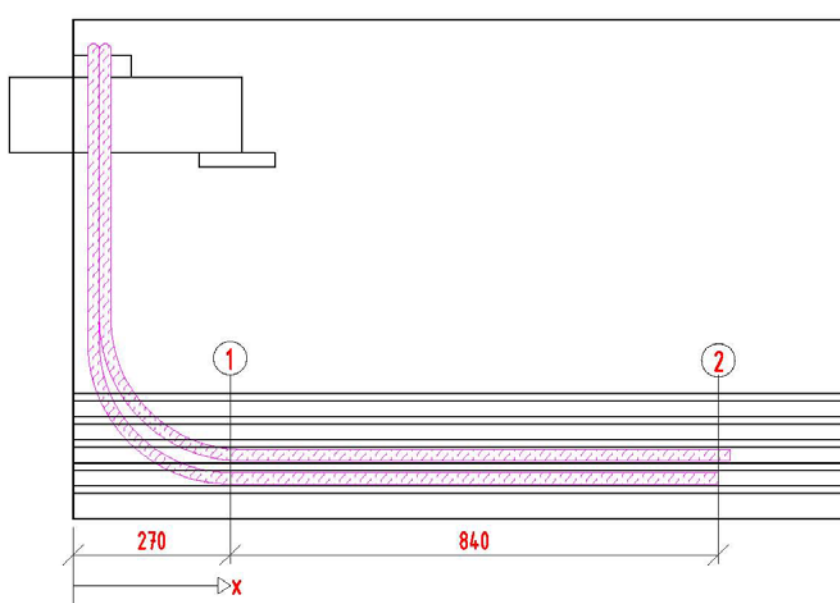
$$\sigma_{pmo} = 1200 \text{MPa}$$

$$\Rightarrow l_{pt} = 1,0 \times 0,19 \times 12,7 \text{mm} \times 1200 \text{MPa} / 2,03 \text{MPa} = 1426 \text{mm}$$

$$\Rightarrow l_{pt1} = 0,8 \times l_{pt} = 0,8 \times 1426 \text{mm} = 1141 \text{mm. To be used in evaluation of local cross section stresses.}$$

$$\Rightarrow l_{pt2} = 1,2 \times l_{pt} = 1,2 \times 1426 \text{mm} = 1711 \text{mm. To be used in evaluation of anchorage. Stress after all losses assumed as } 0,9\sigma_{pmo}. \text{ (10\% loss)}$$

7) Anchoring:



Assuming the horizontal part of the front anchoring bar is 840mm (\approx equals the minimum calculated lap length). I.e. the bar ends at $x=270+840=1110\text{mm}$.

Section 1:

Force anchored in the tendons (5 tendons) at $x=270\text{mm}$:

Assume 10% loss of pre-stressing force:

$$F_{sp1} = 5 \times P \times 0,9 \times 270 \text{mm} / 1711 \text{mm} = 5 \times 120 \text{kN} \times 0,9 \times 270 \text{mm} / 1711 \text{mm} = 85 \text{kN}$$

Force anchored in $\varnothing 16$:

$$F_{\varnothing 16} = 271 \text{kN}$$

Total anchored force:

$$F = F_{sp1} + F_{\varnothing 16} = 85 \text{kN} + 271 = 356 \text{kN}$$

Tension in reinforcement at $x=270\text{mm}$: (clause 6.2.3(7))

$$S(x) = M(x) / z + 0,5 V_{Ed} \times (\cot(\theta) - \cot(\alpha))$$

$$= M(x) / z + 0,5 \times V_{Ed} \times (\cot(45) - \cot(90)) \text{ (assume 45degrees concrete struts and vertical links)}$$

$$= M(x) / z + 0,5 \times V_{Ed} \times (1 - 0)$$

$$= M(x)/z+0,5 \times V_{Ed}$$

Moment at x=270:

$$M(x=270)=180\text{kN} \times (270+75)\text{mm}=62,1\text{kNm}$$

Assume $z=0,9d=0,9 \times 733\text{mm}=660\text{mm}$ (approximately)

$$S(x=270)=62,1\text{kNm}/0,660\text{m}+271\text{kN}/2=230\text{kN}$$

⇒ The anchoring at x=270mm is sufficient.

Section 2:

Force anchored in the tendons (5 tendons) at x=1110mm:

$$F_{sp1}=5 \times P \times 0,9 \times 1205\text{mm}/1711\text{mm}=5 \times 120\text{kN} \times 0,9 \times 1110\text{mm}/1711\text{mm}=350\text{kN}$$

Force anchored in $\varnothing 16$:

$$F_{\varnothing 16}=0\text{kN}$$

Total anchored force:

$$F=F_{sp1}+F_{\varnothing 16}=350\text{kN}+0\text{kN}=350\text{kN}$$

Tension in reinforcement at x=1110mm: (clause 6.2.3(7))

$$S(x)=M(x)/z+0,5V_{Ed} \times (\cot(\theta)-\cot(\alpha))$$

$$=M(x)/z+0,5 \times V_{Ed} \times (\cot(45)-\cot(90)) \text{ (assume 45degrees concrete struts and vertical links)}$$

$$=M(x)/z+0,5 \times V_{Ed} \times (1-0)$$

$$=M(x)/z+0,5 \times V_{Ed}$$

Moment at x=1110:

$$M(x=1110)=180\text{kN} \times (1110+75)\text{mm}=213,3\text{kNm}$$

Assume $z=0,9d=0,9 \times 733\text{mm}=660\text{mm}$ (approximately)

$$S(x=1110)=213,3\text{kNm}/0,660\text{m}+180\text{kN}/2=413\text{kN}$$

⇒ The tendons will not have sufficient anchorage at x=1110mm.

Calculating the point where the tendons are sufficient anchored to carry the load:

Tension in reinforcement at x: $S(x)=180\text{kN} \times (x+75\text{mm})/660\text{mm}+180\text{kN}/2$

Force anchored in the tendons at x:

$$\text{If } x < 1711\text{m: } F(x)=5 \times 120\text{kN} \times 0,9 \times x/1711\text{mm}$$

$$\text{If } x > 1711\text{m: } F(x)=5 \times 120\text{kN} \times 0,9 + \Delta\sigma_{sp}(x) \times A_{sp} \times 5$$

$$\Delta\sigma_{sp}(x)=f_{bpd}/(\alpha_2 \times \varnothing) \times (x-1711) \times 10^{-3}$$

$$\Rightarrow F(x)=5 \times 120\text{kN} \times 0,9 + f_{bpd}/(\alpha_2 \times \varnothing) \times (x-1711) \times 10^{-3} \times A_{sp} \times 5$$

$$\Rightarrow F(x)=540\text{kN} + 2,03\text{MPa}/(0,19 \times 12,7) \times (x-1711) \times 10^{-3} \times 100\text{mm}^2 \times 5$$

$$\Rightarrow F(x)=540+0,420x-720=0,420x-180$$

Try if $x < 1711$:

$$180kN \times (x + 75mm) / 660mm + 180kN / 2 = 540kN \times x / 1711$$

$$0,273x + 110,5 = 0,316x$$

$$0,043x = 110,5$$

$$x = 110,5 / 0,043$$

$$x = 2570mm$$

Calculated $x > 1711 \Rightarrow$ the second expression shall be used:

$$180kN \times (x + 75mm) / 660mm + 180kN / 2 = 0,420x - 180$$

$$0,273x + 110,5 = 0,420x - 180$$

$$0,147x = 290,5$$

$$x = 290,5 / 0,147$$

$$x = 1976mm$$

\Rightarrow The horizontal part of the $\varnothing 16$ anchoring bars has to be extended beyond the calculated required lap length (evaluated in clause 5), and end at $x \geq 1976$.

8) Reinforcement due to splitting stress:

$$A_s = 0,22 \times P_{u1} / f_s$$

$$\Rightarrow A_s = 0,22 \times 5 \times 120000N / 300MPa$$

$$= 440mm^2$$

$$\text{To be located within: } 0,5 \times (l_{pt1} + h_1) = 0,5 \times (1141mm + 863mm) = 1002mm \leq h_1 = 863mm \Rightarrow 863mm$$

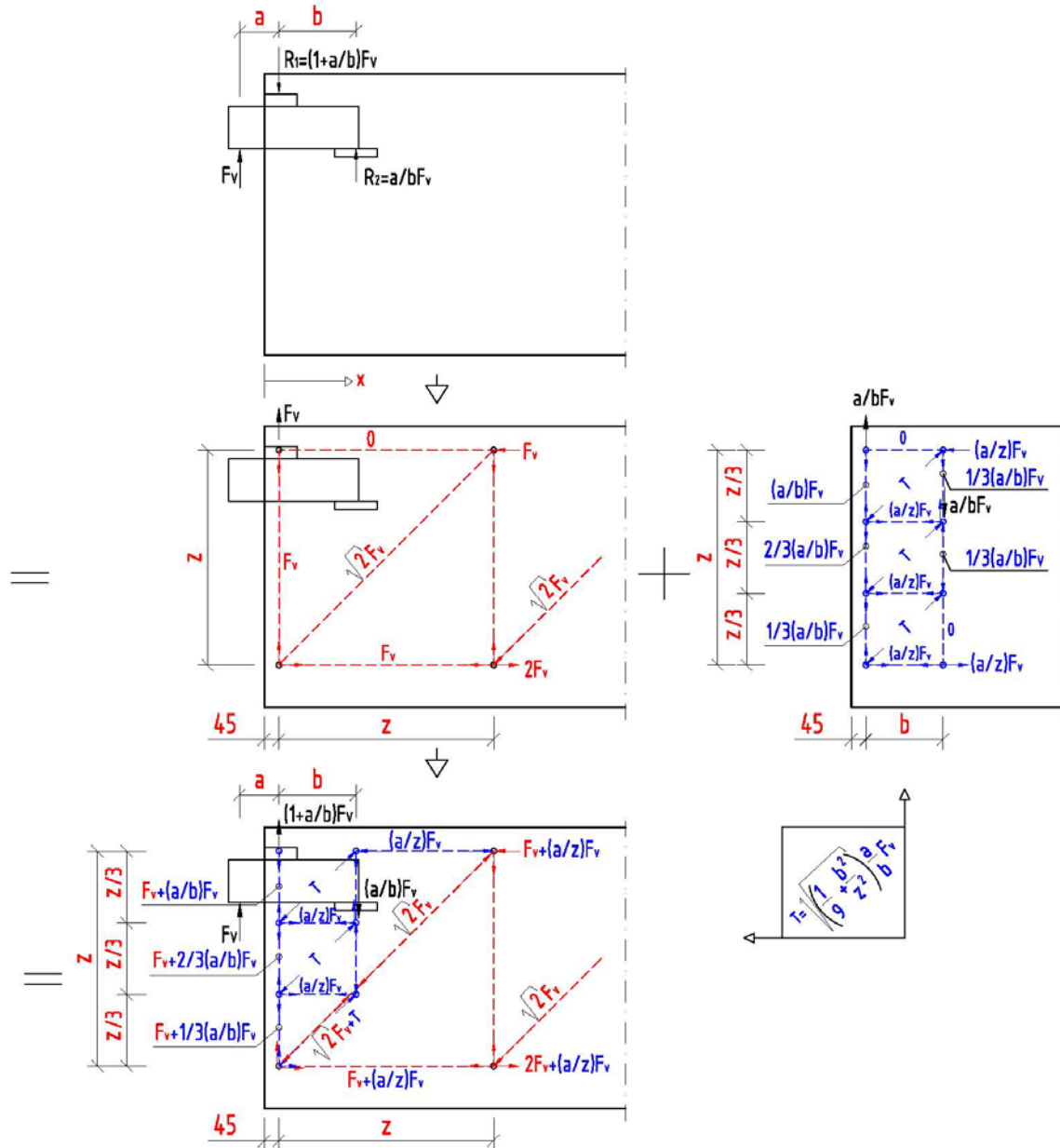
$$\text{Corresponds to: } 440mm^2 / 0,863m = 510mm^2/m$$

9) Links/strut and tie model:

For this particular DT, the height z is approximately equal to $3x_b$. Hence, the bending moment from the cantilevering may be assumed transferred to the main reinforcement of the beam through a local truss with three levels. This is illustrated as a blue truss in the end of the DT, see below Figure. The compression struts in this truss will be angled in approximately 45 degrees.

The support reaction force/shear force itself may be carried by a truss with a height equal to z . This is illustrated as a red truss.

The two trusses are both in equilibrium, and the forces may be superimposed.



Evaluation of local strut and tie model:

- The model implies that only 1/3 of the force R_2 have to be carried by the vertical link at back of the unit. Nevertheless, it is chosen to calculate the required extra reinforcement at this point based on the value of R_2 . (conservative assumption)
- Considering the two horizontal ties towards the end of the DT as smeared, the horizontal force per unit height of the DT becomes: $(a/z) \times F_v / (z/3)$. Where $z/3 \approx b$, hence this corresponds to: $(a/z) \times F_v / b = (a/b) \times F_v / z = R_2 / z$. \Rightarrow Prescribe horizontal bars anchoring this force.

a) Required shear reinforcement $x < 45 + b$:

$$\frac{A_s}{s} = \frac{V_{Ed}}{z \times f_{ywd} \times \cot \theta} = \frac{R_1}{z \times f_{ywd} \times \cot \theta} = \frac{271000N}{0,660m \times 435MPa \times \cot(45)} = 944mm^2 / m$$

Shear compression:

$$V_{Rd} = \frac{\alpha_{cw} \times b_w \times z \times v_1 \times f_{cd}}{(\cot \theta + \tan \theta)}$$

$\alpha_{cw} = 1,0$ (neglecting effect of pre-tensioning)

$b_w = 120mm$

$z = 660mm$

$v_1 = 0,6 \times (1 - f_{ck}/250) = 0,6 \times (1 - 45/250) = 0,492$

$\theta =$ assume 45grader

$$V_{Rd} = \frac{1,0 \times 120mm \times 660mm \times 0,492 \times 25,5MPa}{1 + 1} = 497kN > R_1 \rightarrow OK$$

b) Required shear reinforcement $45 + b < x < 45 + z$:

$$\frac{A_s}{s} = \frac{V_{Ed}}{z \times f_{ywd} \times \cot \theta} = \frac{F_V}{z \times f_{ywd} \times \cot \theta} = \frac{180000N}{0,660m \times 435MPa \times \cot(45)} = 627mm^2 / m$$

Shear compression:

$$V_{Rd} = \frac{\alpha_{cw} \times b_w \times z \times v_1 \times f_{cd}}{(\cot \theta + \tan \theta)}$$

$\alpha_{cw} = 1,0$ (neglecting effect of pre-tensioning)

$b_w = 120mm$

$z = 660mm$

$v_1 = 0,6 \times (1 - f_{ck}/250) = 0,6 \times (1 - 45/250) = 0,492$

$\theta =$ assume 45grader

$$V_{Rd} = \frac{1,0 \times 120mm \times 660mm \times 0,492 \times 25,5MPa}{1 + 1} = 497kN > F_v \rightarrow OK$$

c) Horizontal stirrups at the end of the DT:

$$\frac{A_s}{s} = \frac{R_2}{z \times f_{ywd}} = \frac{91000N}{0,660m \times 435MPa} = 317mm^2 / m$$

Amount of reinforcement below the unit:

$$A_s = 317mm^2 / m \times 2 / 3z = 317mm^2 / m \times 2 / 3 \times 0,66m = 140mm^2$$

\Rightarrow 2 x U-shaped $\emptyset 8$ bars ($A_s = 200mm^2$).

Summary – reinforcement in end of the DT:

The required amount of reinforcement due to splitting stresses is less than the required amount of shear reinforcement calculated in 9a and 9b.

⇒ $X < \approx 282\text{mm}$: Links due to shear force (9a)

$$\frac{A_s}{s} = 944\text{mm}^2 / m$$

Select $\varnothing 8$ stirrups c/c75 (lapped with u-shaped $\varnothing 12$ bars)

$$\frac{A_s}{s} = \frac{\pi \times (4\text{mm})^2 \times 2}{0,075\text{m}} = 1340\text{mm}^2 / m$$

⇒ $282 < X < \approx 705\text{mm}$: Links due to shear force (9b)

$$\frac{A_s}{s} = 627\text{mm}^2 / m$$

Select $\varnothing 8$ stirrups c/c150 (lapped with u-shaped $\varnothing 12$ bars)

$$\frac{A_s}{s} = \frac{\pi \times (4\text{mm})^2 \times 2}{0,150\text{m}} = 670\text{mm}^2 / m$$

$705 < X$: Required shear reinforcement to be evaluated according to shear force distribution in the DT.

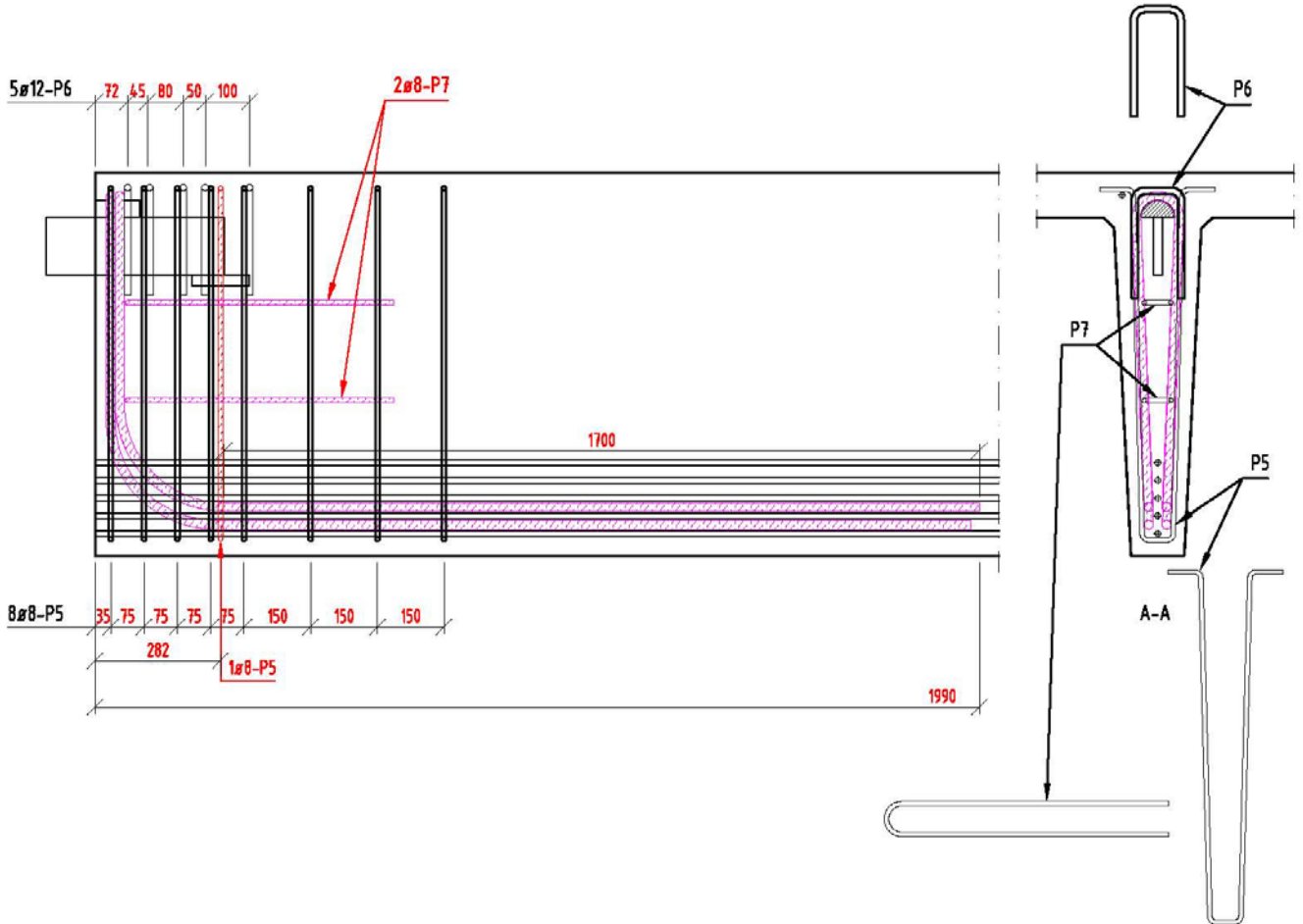
At $X=282$:

Required reinforcement: 210mm^2 . ⇒ 1 extra $\varnothing 8$ (100mm^2) at this location. The remaining 110mm^2 can be carried by the specified distributed links.

Anchoring (7):

⇒ Horizontal part of $\varnothing 16$ anchoring bars in front =1700mm. (Ends at $x > 1976\text{mm}$)

Suggested reinforcement in end of the DT:



REVISION HISTORY	
Date:	Description:
19.09.2013	First Edition.
11.01.2016	Included revision history table. Updated reinforcement quality.
25.05.2016	New template